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Hybrid control on bifurcation for a delayed fractional gene regulatory network

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ABSTRACT

A novel delayed fractional-order gene regulatory network model is proposed in this paper. Firstly, the sum of delays is chosen as the bifurcation parameter, the conditions of the existence for the Hopf bifurcation are obtained by analyzing the associated characteristic equation. Then, it is the first time that a hybrid controller is designed to control the Hopf bifurcation for the proposed network. It is demonstrated that both fractional order and feedback gain can effectively control the dynamics of such network, the stability domain can be extended under the small fractional order or feedback gain. The controller is enough to delay the onset of the Hopf bifurcation and achieve some desirable dynamical behaviors by changing the appropriate order or feedback gain. Finally, numerical simulations are employed to justify the validity of our theoretical analysis.

1. Introduction

Fractional calculus as generalization of ordinary inter-order calculus is a mathematical topic with more than 300 years of history. However, its application to physics, biology and engineering is a recent focus of interest to many researchers. In the past decades, a great deal of researchers found that it has many application in various fields, such as viscoelastic system [1], dielectric polarization [2], electrode–electrolyte polarization [3], electromagnetic waves [4], colored noise [5] and so on. Fractional calculus has many advantages in describing memory and hereditary properties of a host of materials and process in comparison with the classical inter-order ones [6,7]. It reveals that the theory of fractional calculus and fractional-order differential equations can provide a powerful tool in modeling and analyzing phenomenon in the field of system science and engineering.

A large number of genes and gene products, in company with their regulatory interactions, consists of a genetic regulatory network. Analysis indicates genetic regulatory networks are extremely complicated dynamical systems which can represent the interaction functions in gene expression of living cells [8–10]. Genetic regulatory network describes interactions between DNAs, RNAs, proteins and small molecules in an organism, through which gene expression is controlled [11]. In order to gain insight into the regulatory mechanism of gene regulatory networks, it is necessary and important to investigate the dynamics of gene regulatory networks.

Time delays are unavoidable in many real dynamical systems, such as biological systems [12–14], neural networks [15–19] and evolutionary dynamics [20–22]. Hopf bifurcation for two-species delayed Lotka–Volterra competition systems was investigated, the authors found that time delay can destabilize a stable competition system in [12]. In [14], an interesting mathematical model was

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proposed to describe the interaction of inducible defenses and herbivore outbreak, it was illustrated that time delay plays dual effects in the persistence of herbivore populations. In [16], synchronization transitions on scale-free neuronal networks with time delay was studied, the authors found that an appropriate delay can significantly enhance synchronization of excitatory fronts in an intermittent fashion. In [18], the effects of periodic subthreshold pacemaker activity and time-delayed coupling on stochastic resonance over scale-free neuronal networks was considered, it was shown that finite delays in coupling can significantly affect the stochastic resonance on scale-free neuronal networks. In [20], the evolution of cooperation in the spatial public goods game was examined, it was found that intermediate delays enhance network reciprocity. Time delay has an important influence on the dynamic behaviors of the systems, which can lead to complex dynamical behaviors, and an appropriate delay can also improve the stability of the systems [23]. Therefore, it is significant to investigate the time-delayed effect on dynamical behaviors of complex systems theoretically and practically.

Hopf bifurcation analysis is an efficient tool to acquire more information around the equilibrium point of the complex dynamical networks [24,25]. Recently, bifurcation control excert a tremendous fascination on many researchers coming from various disciplines. Customarily, bifurcation control is to design a controller to suppress or reduce some existing bifurcation dynamics for a given nonlinear system, therefore achieving some desirable dynamical behaviors. Various bifurcation control approaches have been proposed [26–28]. It is well known that Hopf bifurcation in delayed inter-order system have been extensively studied, and numerous valuable results have been derived. Due to the development of the fractional calculus, Hopf bifurcation of fractional models have attracted increasing interest in recent years [29–32]. Unfortunately, the impact of time delays on these fractional order models have not been taken into account. Some scholars have made some attempts on the bifurcation of the delayed fractional models lately [33–35]. In [34], the issue of bifurcation and control for a delayed fractional small world network was analyzed, and a delayed feedback controller was applied to delay the onset of Hopf bifurcation by increasing the gain and changing the fractional-order. In [35], the fractional predator–prey model with time delay was introduced, Hopf bifurcation occurs when the delay passes through a sequence of critical values. It should be pointed out that there are very few results discussing the problem of bifurcation and control for fractional system with time delay [34].

From the perspective of theory and experiment, it is vital to model, analyze, and predict the periodic behaviors of genetic regulatory networks. Recently, the dynamics of the delayed genetic regulatory networks have captured increasing attention [36–39]. However, most of these results only deal with the dynamics of integer-order genetic regulatory networks. Magin argued that the activities of the organism can be accurately described by using fractional order derivative [40]. Till now, there few results with respect to fractional gene regulatory network [41–43]. In [41], a new approach based on fractional differential equations to build the genetic regulatory networks from time series data was proposed. It revealed that the proposed mathematical model is more suitable to model genetic regulatory mechanism. In [42], the stability of genetic regulatory networks is considered, some criteria on the Mittag–Leffler stability and generalized Mittag–Leffler stability are established via the fractional Lyapunov method.

Nevertheless, it is mentioning that these existing research work on the dynamics of fractional gene regulatory networks pays little attention to the effect of time delay. Time delay is an essential factor when modelling the genetic networks due to slow biochemical processes such as gene transcription, translation and transportation. Particularly, bifurcation control for delayed fractional genetic regulatory networks has not been investigated before. Motivated by this fact, the problem of the bifurcation and bifurcation control for fractional gene regulatory network with time delays is investigated in the present paper.

The main contribution of the present paper is that we demonstrate for the first time to design a hybrid controller to control the dynamical behaviors for the proposed fractional genetic regulatory networks. This paper presents a theoretical investigation on dynamics for fractional genetic regulatory networks via the proposed hybrid controller. The main attention is paid to discuss the effect of time delay and feedback gain on the stability and bifurcation for the fractional genetic regulatory network. The rest of this paper is organized as follows: In Section 2, some definitions are recalled and the investigated fractional model are presented. In Section 3, the dynamics of uncontrolled delayed fractional genetic regulatory network is analyzed. In Section 4, the hybrid controller is designed to control Hopf bifurcation for the proposed model. To verify our theoretical findings, numerical simulations are carried out by means of two numerical examples in Section 5. The paper is finished by conclusions addressed in Section 6.

2. Preliminaries and model description

There are several definitions of fractional derivatives. The Riemann–Liouville definition and the Caputo definition are commonly used. Since the Caputo derivative only requires initial conditions given by means of integer-order derivative, representing well-understood features of physical situation and making it more applicable to real world problems. Then we propose the fractional gene regulatory model. This paper is based on the Caputo derivative.

Definition 1 ([44]). The fractional integral of order α for a function $f(x)$ is defined as

$$I^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-\tau)^{\alpha-1} f(\tau) d\tau,$$

where $t \geq t_0$, $\alpha > 0$, $\Gamma(\cdot)$ is the Gamma function, $\Gamma(s) = \int_0^{\infty} t^{s-1} e^{-t} ds$.

Definition 2 ([44]). The Caputo fractional-order derivative is defined by

$${}_{C}D^{\alpha}_{t_0,t} f(t) = \frac{1}{\Gamma(m-\alpha)} \int_{0}^{t} (t-\tau)^{m-\alpha-1} f^{(m)}(\tau) d\tau,$$

where $m-1 \leq \alpha < m \in Z^{+}$.