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On a business cycle model with fractional derivative under narrow-band random excitation

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ABSTRACT

This paper analyzes the dynamics of a business cycle model with fractional derivative of order $\alpha(0 < \alpha < 1)$ subject to narrow-band random excitation, in which fractional derivative describes the memory property of the economic variables. Stochastic dynamical system concepts are integrated into the business cycle model for understanding the economic fluctuation. Firstly, the method of multiple scales is applied to derive the model to obtain the approximate analytical solution. Secondly, the effect of economic policy with fractional derivative on the amplitude of the economic fluctuation and the effect on stationary probability density are studied. The results show macroeconomic regulation and control can lower the stable amplitude of economic fluctuation. While in the process of equilibrium state, the amplitude is magnified. Also, the macroeconomic regulation and control improves the stability of the equilibrium state. Thirdly, how externally stochastic perturbation affects the dynamics of the economy system is investigated.

1. Introduction

Economic fluctuation is an important issue of macroeconomics and has been investigated by many scholars in recent years. In 1930, the French physicist Le Corbeiller firstly applied nonlinear dynamics theory to investigate the problem of economic fluctuation. Then, Kaldor [1] and Kalecki [2] constructed a nonlinear model of economic fluctuation to describe the volatility of the economy system. Kaldor's model is similar to the structure of Van-Der-Pol equation. The characteristic of the Kaldor's model is the nonlinear function of saving, which can lead to the emergence of limit cycles. Kalecki proposed a time delay function, which was supposed to cause economical undulation. In 1950s, Goodwin [3] improved Hicks's model and presented five different versions of the nonlinear accelerator model with an investment delay. Puu and Sushko [4] proposed a nonlinear induced investment function to resist economic depression. Matsumoto [5] reexamined Goodwin's model and stated that continuously distributed time delay has a stronger stabilizing effect than fixed time delay and multiple limit cycles may coexist when the stationary state is locally stable.

The research of the effect of economic policy on economy system has been studied in multiple models. Yoshida and Asada [6] investigated the effect of policy lag on the macroeconomic stability and revealed that policy lag contributed to chaotic motion. Yuthana [7] investigated business cycle synchronization and economic policy linkage in the Association of Southeast Asian Nations (ASEAN), and he measured static and dynamic correlations in both macroeconomic variables and policy variables. The empirical results show evidence of synchronization in key macroeconomic variables. Barseghyan et al. [8] explored the implications of the political economy model of Battaglini and Coate [9] for the behavior of fiscal policy over the business cycle. Jean-Pascal [10] constructed a model with explicit foundations where the optimal policies make both employment and output more volatile than in the no intervention case. Benjamin and Johannes [11] used an estimated New Keynesian model to analyze the role of policy risk in explaining and they directly measured risk from aggregate data and find a moderate amount of time-varying policy risk. Giovanni et al. [12] studied an agent-based model that bridges Keynesian theories of demand generation and Schumpeterian theories of technology-fueled economic growth. They used the model to investigate the properties of macroeconomic dynamics and the impact of public policies on supply, demand and the 'fundamentals' of the economy system. Eurilton [13] estimated monetary policy preference parameters under commitment in a timeless perspective in a sticky-price model in which money can potentially play a key role in business cycles. William et al. [14] analyzed the interaction of inflation with the tax code and its contribution to economic fluctuation and they found significant effects operating through the tax on realized nominal capital gains, and the tax on nominal bond income magnifies these effects.

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As is known to us, the decision by human beings has the memory property. Just like buying and selling of stocks, the decision is made by past experience. The macroeconomic regulation and control is made by the economist and politician. Thus, it is reasonable that the economic policy decision-making shows this memory property. That is to say, today's decision is related to the previous decision. Fractional derivative is shown to be suitable to describe this memory property of the variables. Gemant [15] had a pioneering work in this field and he firstly used fractional calculus to investigate the equations of material behavior. Afterwards, many scholars gave deep researches in fractional calculus [16–21]. Rossikhin and Shitikova [22,23] gave us some excellent results on applications of fractional calculus for dynamic problems of mechanics with multiple scale method. Diethelm [24] proposed a predictor-corrector numerical method for the numerical solution of fractional differential equations. Recently, Machado pointed out that, while individual dynamics of each element has integer-order nature, the global dynamics show both integer and fractional dynamics [25]. So, it is practical to study the fractional derivative systems by replacing the integer derivative with the fractional derivative. In recent years, a lot of papers of fractional modeling are also published. Nick Laskin [26] studied a langevin equation of financial assets with fractional derivative and established a probability distribution function of the returns. Wei-ching Chen [27] had an analysis of nonlinear dynamics and chaos in a fractional-order financial system. Marius-F [28] studied a sample parameter switching (PS) methodology for sustaining the stable dynamics of a fractional-order financial chaotic system. Indranil Pan [29] proposed an active control policy design for a fractional-order financial system.

Stochastic perturbations are omnipresent and inevitable. In an economy system, the random factors are natural disaster, climate change and the behaviors of human beings. These random factors can also influence the dynamics of the economy system. Li [30] studied the first-passage failure of a business cycle model under wide-brand random excitation. Li [31] investigated chaos prediction and its control of the Goodwin model under the deterministic and stochastic excitation. The dynamic characteristics can be more complicated when the random excitation and memory properties are considered simultaneously. Spanos and Zeldin [32] presented a frequency-domain method to study the stochastic systems with fractional derivatives. Liu et al. [33] studied the principal resonance responses of single degree of freedom systems with small fractional derivative damping under the narrow-band random parametric excitation by multiple scales method. As further, Chen and Zhu [34] used stochastic averaging method to investigate the nonlinear systems with fractional derivatives under combined harmonic and white noise excitation. Huang [35] investigated the response and stability of a single degree of freedom strongly nonlinear stochastic system with light damping modeled by fractional derivative. Xu and Li [36] proposed a new Lindstedt Pincare method, which can be used to obtain the approximate solution of fractional oscillators under random excitation. Di matteo et al. [37] put forward a new approximated analytical method to obtain the non-stationary response probability density function of the stochastic systems with fractional derivative based on Winner path integral.

This paper is organized as follows. In Section 2, we propose a mathematical model of the business cycle model with fractional derivative under narrow-band random excitation. In Section 3, the multiple scale method is used to obtain the approximate analytical solution of the stochastic economy system with fractional derivative. Finally, in Section 4, an economy system with nonlinear induced investment function is proposed to verify the validity of the analytical method, and the effect of macroeconomic regulation and control and the intensity of random noise on the amplitude and the stationary probability density of the economy system are investigated.

2. Mathematical model

In the literature [3], Goodwin proposed a mathematical model to study business fluctuation. The model of complex economic system is as follows:

$$\delta\eta\ddot{Y}(t) + (\delta + (1-\alpha)\eta)\dot{Y}(t) - \phi(\dot{Y}(t)) + (1-\alpha)Y(t) = O^*(t), \tag{1}$$

$Y(t)$ is the national income, $\phi(\dot{x}(t))$ is the induced investment function, $O*(t)$ is the sum of autonomous consumption and investment, α is the marginal propensity to consume, η is the investment delay, and δ is the reciprocal of a positive adjustment coefficient. We set δ, α, η as constants.

To eliminate$O*(t)$, we set

$$x(t) = Y(t) - O^*(t)/(1-\alpha), \tag{2}$$

Eq. (1) can be written as

$$\delta\eta\ddot{x}(t) + (\delta + (1-\alpha)\eta)\dot{x}(t) - \phi(\dot{x}(t)) + (1-\alpha)\dot{x}(t) = 0. \tag{3}$$

With some transformations, Eq. (2) can be written as

$$\ddot{x}(t) + \beta\dot{x}(t) - \Phi(\dot{x}(t)) + \omega^2 x(t) = 0, \tag{4}$$

where

$$\beta = \frac{\delta + (1-\alpha)\eta}{\delta\eta}, \ \Phi(\dot{x}(t)) = \frac{1}{\delta\eta}\phi(\dot{x}(t)), \omega^2 = \frac{1-\alpha}{\delta\eta}$$

The induced investment function includes the investment from public sector. And this can be viewed as the economic policy to resist economic depression. Considering the memory property of economic policy decision-making, the nonlinear induced investment function [4] can be written as

$$\Phi(\dot{x}(t)) = I(\dot{x}(t)) - \kappa D^{\alpha} x(t), \tag{5}$$

where $\kappa D^{\alpha} x(t)$is the fractional order linear part that represents the realistic fact that governments tend to distribute infrastructure investment counter-cyclically. $I(\dot{x}(t))$ is the nonlinear part of the induced investment function. κ is the reciprocal of an adjustment coefficient, and considering the random factors in the real world, narrow-band bounded noise is brought in model (1) to depict the random factors of macroeconomic regulation and control adjustment. Eq. (1) can be written as

$$\ddot{x}(t) + \beta\dot{x}(t) + \kappa D^{\alpha} x(t) - I(\dot{x}(t)) + \omega^2 x(t) = f\cos(\Omega t + \gamma W(t)), \tag{6}$$

where $f > 0$ is the strength of periodic factor that can influence the economic system like political cycle, climate change etc, Ω is the frequency of the periodic factor, $\gamma W(t)$ is the external stochastic perturbations.

3. Approximate solution

To use the methods of multiple time scales and to obtain the average equation, we introduce a parameter ε, it is a small positive parameter. Then, Eq. (6) can be written as

$$\ddot{x}(t) + \varepsilon\overline{\beta}\dot{x}(t) + \varepsilon\overline{\kappa} D^{\alpha} x(t) - \varepsilon I(\dot{x}(t)) + \overline{\omega}^2 x(t) = \varepsilon\xi(t), \tag{7}$$