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Chaotic behavior of earthquakes induced by a nonlinear magma up flow

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ABSTRACT

This paper considers the dynamics of a modified 1D nonlinear spring-block model for earthquake subjected to the strengths induced by the motion of the tectonic plates and the up flow of magma during volcanism. Based on the multiple time scales method, we establish that after the slip, the fault remains active and the frictions increase with the power of the earthquake. We also obtain in the non-resonance case that the appearing probability of an event decreases with these frictions. In the resonance case, the dynamics of harmonic oscillations show that the rocks constituting the block will fracture or resist to the effects induced by the magma motion. Our analytical investigations are complemented by numerical simulations from which it appears that, for given values of the magma thrust strength magnitude, the friction coefficient, the quadratic and cubic nonlinear parameters, the system exhibits chaotic behavior.

1. Introduction

During the last five decades, various scientists' attention was focused on the understanding of the complex behavior of earthquakes (known as natural phenomena which occur when an active fault breaks [1,2]). Seismologists have analyzed intensively the characteristics of earthquakes with great success [3–5] but clear descriptions of the properties of medium where the driving plates move are still uncompleted. To better understand the complexity of earthquakes, scientists made use of mathematical rupture modeling dynamics and laboratory studies of rocks friction. So far, these two fields remain relatively disconnected and it is still unclear how laboratory discoveries can be better applied in the dynamical models of earthquake faults [1,6]. The first physical model was introduced by Burridge and Knopoff (BK) [7] to describe the propagation of a crack along an earthquake fault. It described the distribution of energy released as a power law of the energy and could be interpreted as an evidence of self-organized criticality. Constituted of several blocks, the BK model also enabled to simulate sequences of slipping events similar to those of real earthquakes. This model was also used to investigate the origin of some power laws that appear in statistics of earthquakes such as the Gutenberg–Richter law [8–15] and clarify the mechanisms underlying the apparent irregularity of seismic events [16]. In order to improve the understanding in this domain, many research works were focused on the study of a single spring-block model [17–19]. In those works, the motion of the blocks was induced by the strengths due to the movement of the tectonic plates. In the present study, we propose a new model that also takes into account the magma thrust strengths that appears during a volcanic activity and which acts as an external force to modify the motion of the block.

The discovery of the dynamical complexity in the uniform 1D BK model of earthquake fault has brought new urgency to some questions about model of seismic sources. One of the pressing questions concerns the role of *elasticity* in the crustal-plane. This elasticity is one of the characteristics of the rocks that constitute the subsoil of the crust earth [2]. In the physical modeling of earthquake dynamics, the rocks elasticity is assumed to be linear [17–22] but some works are done as a nonlinear parameter of the system [16].

The other challenge that has emerged from the recent studies of the earthquakes' dynamics deals with the prediction of the speeds at which ruptures propagate. As far as the single block model is

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concerned, Vasconcelos [17] investigated and showed for the first time that the phase transition from stick-slip motion to creep is discontinuous in the slow driving. He also demonstrated that when the friction characteristic velocity is greater than *0.5*, the block displacement vanishes after the slip event and the block remains seismologically inactive at the end of the earthquake. In the study discussed here, we could wonder whether this critical velocity will change and what type of phase transition will appear in the presence of the magma thrust strength.

Another challenge is related to the understanding of the chaotic behavior of some earthquakes during seism. Previous studies [12,16,23–25] indicated that the completely uniform 1D BK model, with velocity-weakening stick-slip friction, is a deterministically chaotic dynamical system that exhibits a broad range of earthquake-like events. Concerning the one spring-block model, few studies showed the appearance of chaotic behavior within the system when its motion is unidirectional [19,26–29] or bidirectional [18]. Indeed, Ericson *et al* [19] studied the condition under which a single oscillation model coupled with Dieterich–Ruina's rate and state dependent friction can exhibit a chaotic dynamics. They showed that the system undergoes a Hopf bifurcation to a periodic orbit. This periodic orbit changes from a periodic doubling cascade to a strange attractor recognized as broadband noise in the power spectrum. Kostic *et al* [29] investigated the dynamics of the same model submitted to the effect of an external periodic force induced by a passing seismic wave and which directly affects the acceleration of the block. They established in the inter-seismic creep regime that there are transition to quasi-periodic or chaos-like motion due to the precursory creep regime and seismic motion, respectively. It also appears from their study that when the triggered acceleration changes are of longer duration, a reverse transition from inter-seismic to post-seismic creep regime is detected on a larger time scale.

The dynamical behavior of the phenomenological BK model (for few blocks) with time delay, and variation of the friction strength are examined [30]. Research conducted in [30] gave a solid base for the further investigation of complex dynamics since it includes the global dynamical behavior (far from the stationary solution) with heterogeneous initial conditions and different values of spring constants. Kostic et al [31] exploited this BK model to examine the friction memory effect in complex dynamics of earthquakes. Another study showed that the BK model exhibits a deterministic chaotic behavior [32]. Recent research suggests that earthquake ground motion is likely stochastic rather than deterministic [33]. This work deals with the dynamics of strong earthquakes ground motion while it was not explicitly the case in the previous works [30–32]. The stochastic approach exploited to investigate the earthquake ground motion can be seen as a windfall method to study earthquakes as complex phenomena. However, to abandon the deterministic chaotic approach, the stochastic method has to be improved as the authors mentioned in their paper [33]. For example, it must be able to take into account the effects of different types of seismic waves on the ground motion. On the other hand, we know that earthquake ground motion is also made of slow displacements. Therefore, works still have to be done in this deterministic area on the various phenomena occurring during seism.

In the following, we present analytical and numerical studies of some aspects of these problems. Indeed, we propose a modified version of the Vasconcelos model [17] which includes a horizontal nonlinear spring and takes into account the magma thrust strength that appears during a volcanic activity. The nonlinear spring deals with the nonlinear behavior of rock materials elasticity [16]. Our aim is to examine the effects of these two physical quantities on the earthquake dynamics.

This paper is organized as follows. Section 2 presents the modified earthquake model under consideration and derives the associated equation of motion. Section 3 is devoted to the analytical analysis of the equation of motion through the multiple time scales method. These investigations are made for the case where the frequency of the magma thrust strength is closed to that of the system (resonant case) and when these frequencies are far (non-resonant case). In each case, the stability of the system is examined and the impact of both the magma thrust strength and the nonlinear coefficients are measured. In Section 4, numerical integrations of the amplitude equation are carried out to examine the chaotic behavior of our model. Discussion and concluding remarks are given in the last section.

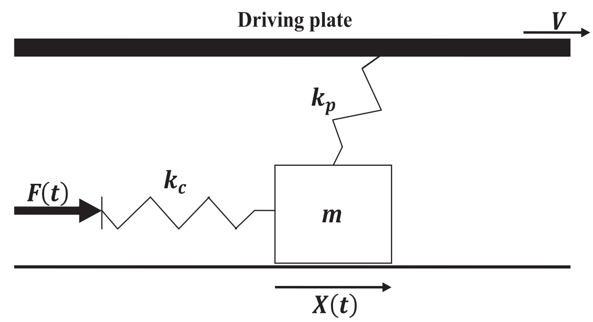

Fig. 1. Spring-block model of earthquakes under consideration.

2. Earthquake model and equation of motion

Fig. 1 depicts the mechanical model of earthquake considered. It consists of a block of mass m attached to a spring of stiffness k_p that moves upper line with a constant velocity V which represents the speed of the tectonic plate. This block rests on a rough surface and is connected by an harmonic spring of stiffness k_c subjected to a force $F(t)$. Fig. 2 enables to see how the magma thrust strength could act. It also shows that one of the two blocks is subjected to the strengths due to the motion of the tectonic plate (driving plate) and that of the thrust of the magma during volcanism outbreak. In this study, we assume that the fracturing zone where the magma outbreaks is orthogonal to the fault [34]. Therefore, the forces created by the thrust of the magma on the earth crust are parallel to the fault during the magma up flow.

Initially ($t = 0$) the system is at rest, and the elastic energy accumulated in horizontal spring k_c is due to the force $F(t)$ that subjects the surrounding fault and the force $f_1(X)$ due to the contraction or dilation of the horizontal spring. The motion of the driving plate will also participate to the movement of the block by exerting a force of the form $f_2(X) = -k_p X(X - Vt)$ [17]. The asperities which exist between the lips generate the frictional force $f(dX/dt)$ opposed to the motion of the block. Therefore, the dynamics of the system of Fig. 1 is governed by the following equation that encloses real variables:

$$m\frac{d^2X}{dt^2} = -k_p(X - Vt) + f_1(X) - f\left(\frac{dX}{dt}\right) + F(t) \quad (1)$$

It is known that the elasticity of rock material is not linear as usually used. To link with the reality, we consider a nonlinear stiffness of rocks and assume that $f_1(X)$ can be explained as follows [16]:

$$f_1(X) = -k_c\left(X + \varepsilon_0\left(\frac{X^2}{a} + \frac{X^3}{2a^2}\right) + 0(\varepsilon_0^2)\right) \quad (2)$$

in which a represents the length of relaxed spring and ε_0 is a small parameter. As any geologist knows, in practice k_c is not the same for large contractions and dilations. Furthermore, the stiffness monotonously decreases when going from contraction to dilation, and then, achieving certain critical value, rock crashes. The