Chaos, Solitons and Fractals 87 (2016) 92–101

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Robust synchronization of impulsively coupled complex dynamical network with delayed nonidentical nodes

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ARTICLE INFO

Article history:
Received 22 October 2015
Revised 24 January 2016
Accepted 8 March 2016
Available online 28 March 2016

Keywords:
Impulsively coupled complex dynamical network
Nonidentical node
Robust synchronization
Hybrid control

ABSTRACT

In this paper, the problem of designing the hybrid control for synchronization of impulsively coupled complex dynamical network is investigated. A general delayed impulsively coupled network with nonidentical nodes and parameter uncertainties is presented. Based on a time varying Lyapunov function associated with the impulsive time sequence and Razumikhin technique, a convex combination technique is employed to obtain the global synchronization criteria in terms of linear matrix inequalities. The derived sufficient conditions for synchronization are closely related to the coupling structure of the network, the lower and upper bound of the adjacent impulsive instant difference, and the impulsive control input strength. The control scheme can be obtained by solving a set of linear matrix inequalities. Typical numerical examples are presented to demonstrate the validity of the theoretical results and the effectiveness of the control strategy.

1. Introduction

As well known, complex dynamical networks are collection of dynamical systems [1]. The complex topological properties can be demonstrated by the connected nodes which are the fundamental units with identical/nonidentical contents. As the major collective behavior, the synchronization of complex systems or chaotic systems has been extensively investigated because there are practical problems to be solved by applying the complex dynamical networks in the real world. For example, in mammalian brains, some neurons have been surveyed to oscillate in synchrony. The absence of such synchronization may cause unconscious physical tension, when goals cannot be recognized

nor worked toward because the members are 'out of sync' [2–4]. In addition, it has been reported that synchronization of two chaotic systems is possible and has potential practical applications in secure communications [5]. Several computer networks also can be illustrated and analyzed by complex dynamical networks, such as the World Wide Web, food web, power grid, telephone call graphs, etc.

In order to obtain synchronization of complex dynamical networks with continuous coupling, various control methods, including adaptive control [6–8], pinning control [9,10], sampled-data control [11,12], intermittent control [13,14], and output synchronization [15] have been proposed to synchronize the complex dynamical networks. Recently, impulsive control method has been widely used to describe the real phenomenon, such as pest control in agriculture [21], the impulsive HIV dynamics [22,23], the diabetic patient model [22], and predator-prey model with stage-structure [24], etc. As discussed in [21], integrated pest management (IPM) is a sustainable method in pest control in agriculture. The key factor of utilizing IPM is a comprehension of the ecology of the cropping system, including that of the pest, their natural enemies, the natural environment and the inter-relationships. Simultaneously, the human actions including spraying pesticides, releasing natural enemies and infected pests in IPM can be achieved instantaneously or within a short period. Therefore, the impulsive control method can be used to describe the human action and determine the optimal time of the implementation of human actions in the IPM. The physical insight of the impulsive control method is the sudden change or the 'jump' of the system states. Additionally, the implementation of the impulsive effects turns out to be very simple and convenient in these systems. For instance, in the integrated pest management (IPM), to spray pesticides at the optimal time can be seen as the implementation of the impulsive effects [21]. In the Bergman's model [22], to inject insulin at certain time to maintain the glucose level also can be viewed as the implementation of the impulsive effects.

Because of the effectiveness, economy and convenience of the implementation, the impulsive control strategy [16–20] has been

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http://dx.doi.org/10.1016/j.chaos.2016.03.010

widely designed to address the synchronization issue of the complex dynamical networks. For instance, by employing the concepts of both joint connectivity and sequential connectivity [18], the complex dynamical networks can be synchronized even if the topology is not connected at any time instant. Using the concept of average impulsive interval, new synchronization criteria via impulsive control strategy have been derived [19] for switched neural networks with unbounded distributed delays. Based on the theory of impulsive functional differential equations and linear matrix inequality method, dual-stage impulsive control method [20] has been proposed to guarantee the synchronization of a class of chaotic delayed neural networks with parametric uncertainties. Particularly, in order to avoid the implementation difficulty of the controller, the control topology in terms of impulsive distributed control has been proposed [25] to synchronize the complex networks with continuous coupling. Since then, the applications of the control topology along with impulsive distributed control method have further developed [26–30], and reference therein.

In the view of practice, several systems are coupled at fixed discrete instants only, such as the species-food model in biology, the information transfer and exchange in ants, and the model of integrated circuit [36]. These systems are coupled at discrete instants through impulsive connections , rather than coupled with continuous connections. As a consequence, investigation on the synchronization for the impulsively coupled systems has received considerable attention [31–40]. A model for impulsively coupled systems has been first proposed in [36], and sufficient conditions for synchronization of dynamical networks have been derived. Robust delay-independent synchronization criteria have been obtained [38] for a general uncertain impulsively coupled switched network with time delays. By employing the partial contraction theory of impulsive systems and by studying the contraction analysis of impulsive coupled oscillators, general criteria have been presented [39] for synchronization of impulsive coupled oscillators. As a typical class of impulsively coupled system, the synchronization/consensus problems of multi-agent/multibody systems with impulsive control have been widely investigated in [31–35]. For instance, based on the local information of agent, an impulsive control protocol is designed for consensus of multi-agent systems [35] with/without switching topologies. The problem of output consensus is studied [32] by impulsive control method for multi-agent systems with passive system agents. A procedure for investigating impulsive synchronization motion is given in [34] to synchronize the networked multibody systems described by Lagrange dynamics. Clearly, the impulsive effects in [25–35] can be viewed as synchronizing impulsive effects, and the impulses in [36–40] are desynchronizing impulsive effects.

Clearly, one can observe that the nodes in the impulsively coupled networks discussed in [25–27,30–40] are identical. However, in biology and engineering networks, it is unreasonable to assume that all nodes are identical in a complex dynamical network because individuals inside a network usually have different physical parameters [41]. For example, the nodes in the genetic oscillator networks, even in the same species, are usually nonidentical because of both heterogeneous nutrition status and fluctuated circumstances [13]. Clearly, the synchronization criteria for complex dynamical networks with identical nodes cannot be used to guarantee the synchronization of the complex dynamical networks with nonidentical nodes. Consequently, studying new synchronization criteria for networks with nonidentical nodes has received much attention, and several results have been reported in [1,13,42–45] and references therein. However, in the existing literature, few investigation has been involved to obtain the synchronization criteria for impulsive coupled complex networks with nonidentical nodes under desynchronizing impulses.

In view of this, the synchronization problem of delayed impulsively coupled network with nonidentical nodes is investigated in this paper. The aim is to design a hybrid control strategy to exponentially synchronize the impulsively coupled networks with parameter uncertainties. Inspiring by the method developed in [29,46], a time varying Lyapunov function associated with impulsive time sequence and a convex combination technique are employed to obtain the synchronization criteria in terms of linear matrix inequalities. The local impulsive control gain can be obtain by solving the corresponding linear matrix inequalities. In addition, the results in this paper can be used to guarantee the synchronization of impulsively coupled network without any control input. Numerical examples are given to illustrate that the impulsively coupled network with nonidentical nodes can be synchronized by the proposed control scheme effectively. The rest of this paper is organized as follows. Some notations, definitions, lemmas are introduced in Section 2. The synchronization problem of impulsively coupled network is investigated in Section 3. Typical numerical examples are presented in Section 4. The conclusion is finally drawn in Section 5.

2. Preliminary

Notation: Let $\mathbb{R}$ denote the set of real numbers, $\mathbb{Z}_+$ denote the set of positive integers, and $\mathbb{R}^n$ denote the n-dimensional real space with the Euclidean norm $\|\cdot\|$. $\otimes$ represents the Kronecker product. $M > 0(<0)$ denotes that the matrix M is symmetric positive(negative) definite. The notations M^T and M^{-1} mean the transpose of M and the inverse of a nonsingular matrix M, respectively. $\lambda(M)$ represents the eigenvalue of the matrix M. $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ represent the minimal eigen value and the maximal eigen value of the symmetrical matrix M, respectively. I denotes the identity matrix of appropriate dimension. $\Lambda = \{1,\dots,N\}$ and $diag_{i\in\Lambda}\{M_i\} = diag\{M_1,\dots,M_N\}$. Let the notation $\star$ denote the symmetric block in a symmetric matrix. For any interval $J \subseteq \mathbb{R}$, $PC = \{\psi : J \to \mathbb{R}^n$ is continuous everywhere except at finite number of point t, where $\psi(t^+)$, $\psi(t^-)$ exist and $\psi(t^+) = \psi(t)\}$. For $\varsigma \in PC([-\tau,0],\mathbb{R}^n)$, define $\|\varsigma\| = \sup_{-\tau\le s\le 0}\|\varsigma(s)\|$.

Consider the delayed impulsively coupled complex dynamical network with N nonidentical nodes and uncertainties as following

$$\begin{cases} \dot{x}_i = D_i x_i(t) + \tilde{A}_i f_i(x_i(t)) + \tilde{A}_{id} f_i(x_i(t-\tau_i(t))) + u_{ic}(t), t \neq t_k, \\ \Delta x_i(t_k) = C_k\left(\sum_{j=1}^{N} g_{ij} x_j(t_k^-)\right) + u_{id}(t_k^-), \ t = t_k, \\ x_i(s) = \phi_i(s), \ s \in [-\tau, 0], \end{cases} \quad (1)$$

where $i, j \in \Lambda$, $x_i(t) \in \mathbb{R}^n$ is the state vector, $f_i(x_i(t))$ is a general continuous vector function. $\tau_i(t)$ is a time delay function with $0 \le \tau_i(t) \le \tau$, where $\tau > 0$ is a real scalar. $\tilde{A}_i = A_i + \Delta A_i$ and $\tilde{A}_{id} = A_{id} + \Delta A_{id}$ are real matrices with uncertainties. C_k is a $n \times n$ impulsive matrix. $G = (g_{ij})_{N\times N}$ denotes the coupling matrix. $g_{ij} = g_{ji} > 0$ if there exists a connection between the node i and the node j $(j \neq i)$, otherwise $g_{ij} = g_{ji} = 0$. For the dynamical network (1), it requires that $g_{ii} = -\sum_{j=1,\ j\neq i}^{N} g_{ij}$.

The impulsive time sequence $\{t_k\}_{k\ge 1}$ satisfies $t_0 < t_1 < t_2 < \cdots$ with $\lim_{k\to\infty} t_k = \infty$ and $x(t_k^-) = \lim_{h\to 0^-} x(t_k + h)$. There exist constants $\delta_1 > 0$ and $\delta_2 > 0$ such that $\delta_1 \le t_{k+1} - t_k \le \delta_2$ for $k \in \mathbb{Z}_+$.

Remark 1. In the impulsively coupled network (1), each node can have different time varying delay and different node dynamics. Consequently, the model in this paper can represent variety of dynamical networks. Particularly, if the function $f_i = f$, the matrices $D_i = D$, $\tilde{A}_i = \tilde{A}$, and $\tilde{A}_{id} = \tilde{A}_d$ for all $i \in \Lambda$, the synchronization problem of impulsively coupled network (1) can be viewed as the