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Effect of toxic substance on delayed competitive allelopathic phytoplankton system with varying parameters through stability and bifurcation analysis

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ABSTRACT

We have studied the combined effect of toxicant and fluctuation of the biological parameters on the dynamical behaviors of a delayed two-species competitive system with imprecise biological parameters. Due to the global increase of harmful phytoplankton blooms, the study of dynamic interactions between two competing phytoplankton species in the presence of toxic substances is an active field of research now days. The ordinary mathematical formulation of models for two competing phytoplankton species, when one or both the species liberate toxic substances, is unable to capture the oscillatory and highly variable growth of phytoplankton populations. The deterministic model never predicts the sudden localized behavior of certain species. These obstacles of mathematical modeling can be overcomed if we include interval variability of biological parameters in our modeling approach. In this investigation, we construct imprecise models of allelopathic interactions between two competing phytoplankton species as a parametric differential equation model. We incorporate the effect of toxicant on the species in two different cases known as toxic inhibition and toxic stimulatory system. We have discussed the existence of various equilibrium points and stability of the system at these equilibrium points. In case of toxic stimulatory system, the delay model exhibits a stable limit cycle oscillation. Analytical findings are supported through exhaustive numerical simulations.

1. Introduction

Environmental pollution is a threatening problem for the industrialized society in the modern era because it leads to damage of both terrestrial and aquatic environments. Various kind of industrial discharges and chemical wastes are polluting the air and contaminating the streams, lakes, rivers and oceans with varieties of toxicant and chemicals such as arsenic, lead, cadmium, zinc, copper, iron, mercury etc. [1–3]). A huge amount of toxicant and contaminants cross the threshold into the ecosystem one by one which is very dangerous to the exposed population as well as human beings also. There are many species which becomes extinct and several others are at the verge of extinction because of uncontrolled effects of toxicant to the environment. Therefore, the study of the effects of the toxic substances on ecological communities is becoming quite important from both environmental and conservation point of view. Almost every aquatic life is based upon plankton. They freely float and weakly swim near the surface of most aquatic environments namely, lakes rivers, estuaries and oceans [4]. Phytoplankton is the plant from plankton community capable of photosynthesis in presence of sunlight; algae is the most common form of phytoplankton. Phytoplankton serves as a basic food resource for many aquatic animals. During photosynthesis they release oxygen and absorb carbondioxide from the surrounding of the environment. The most interesting features of phytoplankton populations are rapid cell propagation. As a result, population has rapidly increased of their biomass. After a certain time of period, population almost has rapidly decreased of their biomass which referred to as “bloom”. During past two decades there has been a frightening increase in harmful algal blooms (HAB) of both toxic as well as non toxic species ([5–7]). As HAB has negative impact on environment as well as ecology, researchers have given special attention towards HAB. Researchers investigated to understand the bio-physical reason behind harmful toxic blooms but till now they could not find accepted reasons behind this phenomenon. The variations of phytoplankton densities within a marine environment is affected by many important external factors such as variation of

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essential nutrients environment forcing due seasonal changes etc. [8]. On the country, allelopathy is an important factor affecting the phytoplankton growth. Our investigation in this paper is mainly emphasized on allelopathy. Molisch [9] first introduced the term allelopathy in 1937 after that Rice [9] applied extensively to phytoplankton communities. Allelopathy is the effect of one species on the growth of another through the release of chemical compounds into the surrounding of the environment. This release of chemical compounds is known as “allelochemicals”.

In early 1980s a new field was introduced in the mathematical models which was composed with the effect of a toxicant on a population. The problem of estimating qualitatively the effect of toxic substances on the species by using mathematical models is a very effective way. The deterministic dynamic model with the effect of toxic substances on various ecosystems were analyzed by Hallam et al. [10,11], Hallam and De Luna [12], De Luna and Hallam [13], Freedman and Shukla [14], Ghosh et al. [15], He and Wang [16], Das et al. [17] and many others. Maynard-Smith [18] first studied mathematical models for the allelopathic interaction between two competing species. He considered two species Lotka-Volterra type competition with an additional term representing the effect of toxic substances released by one species negatively impact the second species. Several researchers [19–22] analyzed different types of mathematical models based on Maynard-Smith model. Allelochemicals often have inhibitory effect on some species (HAB). Another important observation made by researchers is that the increased population of one species might affect the growth of another species or of several other species through the production of allelopathic toxins or stimulators, thus influencing seasonal succession [9]. The toxin produced by the unicellular green alga, chlorella Vulgaris, is an autotoxin that limits the size of its own population [23,24] as well as inhibits the growth of the planktonic algae Asterionella formosa and Nitzschia frustrum (Bacillareae) [25]. Several researchers also observed toxic inhibition of phytoplankton by other phytoplankton as well as some of these algae produce auxins which stimulate the growth of the other algae [26]. Such allelopathic stimulators and inhibitors certainly affect algal succession, blooms and pulses by causing stimulated (inhibited) species to have a selective advantage (disadvantage) in competition [9,27]. It has been suggested by Rice [9] that 'all meaningful' functional ecological models will eventually have to include a category on allelopathic and allelochemic effects.

In nature, time delays occur in almost every biological situation [28] and is assumed to be one of the causes of regular fluctuations on population biomass. In population dynamics, a time delay is introduced when the rate of change of population biomass is not only a function of the present population biomass, but also depends on the past population biomass. Therefore, time delay can be incorporated in the mathematical population model due to various ways such as maturation time, capturing time and other reasons. Moreover, existence of time delays is frequently a source of instability in some way. Many researchers [29–39] have introduced time delay in their respective biological models to make it more realistic. Mukhopadhyay et al. [21] modified the Maynard-Smith [18] to a delay differential equation model. They considered discrete time lag required for maturity of the species to produce toxic subtances. The authors find that there are no destabilizing effects on the dynamics of the model system due to discrete time lag. Abbas et al. [40] extended the Mukhopadhyay et al. [21] model and obtained almost time periodic solution of the model system.

It is well known that the deterministic models having simple dynamic behavior are capable to display a wide range of complex dynamic behavior including chaos when the parameters are periodically perturbed. As plankton models are considered in an open aquatic environment like ponds, lake and oceans, time depending environmental conditions, namely water temperature, salinity, intensity of sunlight, turbulence and thermocline depth are important factors and results in variation of system parameters.

Most of the researches in theoretical ecology have considered models based on the assumption that the biological parameters are precisely known. But in real world, the values of all the biological parameters can not be known precisely for the lack of information, lack of data, mistakes done in the measurement process and determining the initial conditions. To overcome these difficulties, imprecise model is more realistic in the field of mathematical biology. The impreciseness of the bio-mathematical model also occurs due to environmental fluctuations or due to imprecise biological phenomenon. There are several approaches to handle such models having imprecise parameters such as stochastic approach, fuzzy approach, fuzzy stochastic approach etc. Some researchers have introduced fuzzy models in predator-prey system such as Bassanezi et al. [41], Peixoto et al. [42], Guo et al. [43], etc. Again stochastic prey-predator model enormously studied by Abundo [44], Rudnicki [45], Liu et al. [46] and the references therein. However, there are some difficulties to handle the imprecise biological parameters by fuzzy approach as well as stochastic approach. In fuzzy approach the imprecise parameters are replaced by fuzzy sets with known membership function by fuzzy numbers, but it is very difficult to construct a suitable membership function for the imprecise biological parameters. In stochastic approach, the imprecise parameters are assumed to be a random variable with known probability distributions. But to identify the suitable probability distribution for a stochastic approach is very tough. Keeping in minds such difficulties Pal et al. [47–51] first introduced the concept of interval number to present imprecise prey-predator model and use parametric functional form of interval number to illustrate different aspects of the model.

In this paper we have modified the basic mathematical model for allelopathy in the presence of two phytoplankton species. We have extended the deterministic model of Maynard-Smith [18] for two competing species with one species allelopathic to other, to two new delay imprecise models (toxic inhibitory, toxic stimulatory) by taking the biological parameters as interval number. A discrete time lag is introduced in to the model system due to maturity of first species to take part in the interaction between species and also to produce toxic substances. We present the interval numbers in parametric function form and study the parametric model. The dynamical behavior of the parametric model is investigated for different values of the parameter.

2. Prerequisites mathematics

In this section we discuss some basic definitions of the interval number and interval-valued function [52] which have been used to study the imprecise competition delay model.

Definition 1 (Interval number)**.** An interval number B is represented by closed interval $[b_l, b_u]$ and defined by $B = [b_l, b_u] = \{x : b_l \le x \le b_u, x \in \Re\}$, where $\Re$ is the set of real numbers and b_l , b_u are the lower and upper limits of the interval number respectively.

So, every real number can also be presented by the interval number $[b, b]$, for all $b \in \Re$.

Definition 2 (Interval-valued function)**.** Let us consider the interval $[a, b]$ where $a > 0$. The interval $[a, b]$ can be expressed as a function $\phi(p) = a^{1-p}b^p$ for $p \in [0, 1]$. This function is called interval-valued function.

Now we define some arithmetic operations on interval-valued functions. Let $A = [a_l, a_u]$ and $B = [b_l, b_u]$ be two interval numbers.

Addition: $A + B = [a_l, a_u] + [b_l, b_u] = [a_l + b_l, a_u + b_u]$. The interval-valued function for the interval number $A + B$ is given by $\phi(p) = a_L^{1-p} a_U^p$ where $a_L = a_l + b_l$, and $a_U = a_u + b_u$.