Chaos, Solitons and Fractals 87 (2016) 136–145

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Review

Shape synchronization control for three-dimensional chaotic systems☆

Yuanyuan Huang[a,b,*], Yinhe Wang[a], Haoguang Chen[a], Siying Zhang[c]

[a] College of Automation Guangdong University of Technology, Guangzhou 510006, China
[b] College of Computer and Communication Engineering, Changsha University of Science & Technology, Changsha 410016, China
[c] College of Information Science and Engineering, Northeastern University, Shenyang 110819, China

ARTICLE INFO

Article history:
Received 5 August 2015
Revised 14 March 2016
Accepted 17 March 2016
Available online 4 April 2016

Keywords:
Three-dimensional chaotic system
Shape synchronization
Secure communication

ABSTRACT

This paper aims to the three-dimensional continuous chaotic system and shape of the chaotic attractor by utilizing the basic theory of plane curves in classical differential geometry, the continuous controller is synthesized for the master–slave synchronization in shape. This means that the slave system can possess the same shape of state trajectory with the master system via the continuous controller. The continuous controller is composed of three sub-controllers, which respectively correspond to the master–slave synchronization in shape for the three projective curves of the chaotic attractor onto the three coordinate planes. Moreover, the proposed shape synchronization technique as well as application of control scheme to secure communication is also demonstrated in this paper, where numerical simulation results show the proposed control method works well.

1. Introduction

During the past twenty years, there has been a growing interest in the synchronization of master-salve for chaotic systems in the field of nonlinear scientific research since the ground breaking researches by Pecora and Carroll [1] in1990. Due to the various master-salve variables to be chosen, the different schemes for master–slave synchronization of chaotic systems are proposed. The most important type of synchronization is complete synchronization [2], generalized synchronization [3,4], phase synchronization [5–7], Lag synchronization [8,9], and even anti-synchronization [10].

The master–slave synchronization of chaotic systems is believed to have a great variety application in secure communication [11–13]. In most of the communication schemes, a signal generated by the master system may be used as an input of the slave system for achieving synchronization. Actually, three main message encoding schemes were developed: chaotic masking [14], chaotic shift keying [15], and chaos modulation [16], where chaotic masking is one of the first and simplest techniques for transmitting information in a secure fashion.

From the aforementioned research, chaotic synchronization is a key technology in the secure communication and many researchers also utilized control theories to improve chaotic synchronization. It is observed that the above mentioned types of chaotic synchronization have been proposed from the distance between state variables of master and slave systems, which did not consider the shape of the chaotic attractor. For example, if two chaotic systems are in state of complete synchronization implies that the distance between states of both systems tends to zero in time; slave and master systems are said to be in state of generalized synchronization if the distance between states of both systems meets a specific function. As well known, a chaotic system is often described by the smoothly dynamical differential equations; so its state trajectory (curve) can be regarded as the chaotic attractor in some bounded region in the phase space when the initial condition is given. In macroscopic view, chaotic attractor means a curve consisting of trajectory points in phase space. Each chaotic system has its own unique chaotic attractor, which showing the same or different shapes in phase space with different positions. According to the differential geometry theory of space curve, the chaotic attractors with the same shape and different positions in phase space may be coincident with each other via the suitable rigid body motions.

Inspired by the above-mentioned idea, based on differential geometry theory, this paper addresses the shape synchronization of master and slave chaotic attractors via control scheme for three-dimensional chaotic systems. A novel master–slave synchronization in shape may be proposed how to design a suitable control scheme for the slave system, it can be synchronized in the shape with the

☆ This work was supported in part by the National Science Foundation of China 61273219; and the Specialized Research Fund for the Doctoral Program of Higher Education 2013-4420110003.

* Corresponding author. Tel.: +8613873115443.
E-mail addresses: snailhyy@126.com (Y. Huang), yinhewang@sina.com (Y. Wang), ap0604302@126.com (H. Chen), Siyingz@sina.com (S. Zhang).

http://dx.doi.org/10.1016/j.chaos.2016.03.025
0960-0779/

master-chaotic system (this is called as the shape synchronization). In other word, the shape synchronization means that the shape of the salve system's state trajectory curve is asymptotically convergent to the shape of the master system's chaotic attractor with the passage of time via control scheme. Compared with other chaotic synchronization, the shape synchronization focus far more on the shape and is not directly related to the distances between state variables of the master and slave systems.

The shape is a significant feature representing the content of the target object. Recently, the relevant shape researches have been widely applied in intelligent transportation, image retrievable, remote sensing image, target detection identification and medical images so on. Such as shape representation and shape matching are very important problem in computer vision and image processing in ref. [17,18]. Automatic lip-synchronization technique [19] is also hot pints, which the computer-based facial animation can be automatically tackled from the identify lip shapes for a given speed sequence by using speech analysis. The shape synchronization is also one of the most remarkable phenomena found in biological systems, which means that biological species present similar shape characteristics because of the same living environment, even in evolutionarily distant organisms [20,21]. Therefore, the research on chaotic shape synchronization is worth considering. To the author's best knowledge, very limited works have been probed into shape synchronization for chaotic systems. In recent work [22,23], we proposed shape synchronization for a class of two-dimensional chaotic systems and its application in secure communication system. The method uses the fundamental theorem on plane curves to design the continuous controller for shape synchronization. In proposed secure communication system, the shape characteristics of master's attractor as synchronized signals are sent through the public channel, which can be reproduced in receiving end via the phase space reconstruction method. Our prior work was mainly done in two-dimensional chaotic systems. Indeed, there are a large number of three-dimensional systems, such as the Lorenz System [24], the Rössler System [25], Chua system [26], Colpitts System [27], and Rititake System [28]. In this paper, we continue to study shape synchronization of three-dimensional chaotic systems in the base of the former work.

We have two goals in this paper. The first goal is to realize the master–slave synchronization in shape for a class of three-dimensional continuous systems of chaos. In order to achieve shape synchronization, the first step is to separately project the master-chaotic attractor onto the coordinate planes *xoy, yox, xoz* and obtain three projective chaotic systems. The second step is to design appropriate salve systems to be coincident with the three projective chaotic systems via control scheme, respectively. Finally, the master–slave shape synchronization is achieved via reconstruction of the chaotic attractor of master system. The second goal is to propose a secure communication scheme based on shape synchronization. The scheme puts the shape characteristics variables of master-chaotic system made available as the synchronized signals, which are transmitted via the public channel. Based on principle of chaotic masking, the three-dimension information signal (be decomposed into three components), is to be encrypted in the chaotic signals of master-chaotic system. The controller design is used to achieve the shape synchronization between the transmitter and receiver in the scheme. Thus, it ensures the information signal encrypted by the master-chaotic system in the transmitter can be recovered in the receiver via a so-called shape synchronization controllers and high performance communication can be obtained.

This organization of the work is as follows. Section 2 briefly describes the basic concepts of classical differential geometry are given. Section 3 outlines the systems under study and theory model. In Section 4, the definition of shape synchronization is presented and the process of design of shape synchronization controllers is described in detail. This is then followed by the problem of reconstruction of the overall slave system shape synchronized with a three-dimensional chaotic system in Section 5. The following section we proposed the chaotic secure communication scheme based on shape synchronization. Numerical simulations and secure analysis to show effectiveness of the proposed scheme is also given in this section. Conclusion comes in the last section.

2. The basic concepts of classical differential geometry for plane curve

2.1. Curvature of a parametric curve

A parametric curve in R^n is a smooth mapping $r: I\subset R\rightarrow R^n (r=r(t))$. When $n=2$, it is called a plane curve and when $n=3$, it is called spatial curve. The parametric curve r is said to be regular if $r'(t)=dr(t)/dt$ never vanishes on I. From now on, $r=r(t)$ denotes a regular parameterized curve.

The arc-length s from the point $r(t_0)$, $t_0\in I$, to a given point $r(t_1)$, $t_1\in I$, is defined by $s(t_1)=\int_{t_0}^{t_1}\|r'(t)\|dt$. When, the curve is regular, $s=s(t)$ is strictly increasing, and has therefore an inverse $t=t(s)$.

Let $r=r(t)=(x(t),y(t))^T$ be a t-parametric plane curve of R^2 (Cartesian right hand Frame $\{O;\, e_1,\, e_2\}$), $t\in R^+=[0,+\infty)$ be the t parameter, and $r=r(t)$ be class c^2 smooth regular curve. Then, the unit tangent vector T may be written as:

$$T(t)=\frac{r'(t)}{|r'(t)|}=\frac{1}{\sqrt{x'^2(t)+y'^2(t)}}\begin{bmatrix}x'(t)\\ y'(t)\end{bmatrix} \tag{1}$$

The unit normal vector N takes the form:

$$N(t)=\frac{1}{\sqrt{x'^2(t)+y'^2(t)}}\begin{bmatrix}-y'(t)\\ x'(t)\end{bmatrix} \tag{2}$$

It is easily seen that $\|T(t)\|=1$, $\|N(t)\|=1$. The vectors T and N are orthogonal ($\langle T(t),N(t)\rangle=0$) for any $t\in R^+$, where $\|\cdot\|$ and $\langle\cdot\rangle$ denote Euclidean norm and inner product, respectively.

There exists a function $\kappa(s)$ such that $\frac{dT(s)}{ds}=\kappa(s)N(s)$, which is called the signed curvature of the curve at the point $r(s)$. The signed curvature at a point also corresponds to variation of the angle $\theta=\theta(s)$ between tangent vector and e_1 with respect to the arc-length:

$$\kappa(s)=d\theta(s)/ds \tag{3}$$

For a plane curve given parametrically in Cartesian coordinates as $r(t)=(x(t),y(t))^T$, the signed curvature is given that

$$\kappa(t)=\frac{x'(t)y''(t)-x''(t)y'(t)}{[x'^2(t)+y'^2(t)]^{\frac{3}{2}}} \tag{4}$$

2.2. Frenet formulas for plane curves

In differential geometry for plane curves, the Frenet formulas describe the kinematic properties of a particle which moves along a continuous, differentiable curve in Euclidean space, it is given as:

$$\begin{aligned}T'(s)&=\kappa(s)N(s)\\ N'(s)&=-\kappa(s)T(s)\end{aligned} \tag{5}$$

Fundamental *Theorem* of *Planar Curves*[29,30]: Suppose $r=r_1(s)$ and $r=r_2(s)$ are regular parameterized curve on the plane R^2, which are parameterized by the arc-length s. If the signed curvature of two curves are not zero everywhere, and are equivalent $(\kappa_1(s)=\kappa_2(s))$, then $r_1(s)$ differs from $r_2(s)$ by a translation followed by a rotation. That is, we have that $r_1(s)=Ar_2(s)+Y_0$, where A is a matrix of the form $A=\begin{pmatrix}\cos\theta_0 & -\sin\theta_0\\ \sin\theta_0 & \cos\theta_0\end{pmatrix}$, which gives