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Study of bifurcations and chaos in the Muthuswamy–Chua system

Imad Al-Deen Hussein A. Al-Saidi*, Furat A. Al-Saymari

Department of Physics, College of Education for Pure Sciences, University of Basrah, Basrah, Iraq

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ABSTRACT

The dynamical behavior of an autonomous simplest electronic circuit that consists of three elements connected in series governed by three ordinary differential equations was investigated. The circuit elements are a linear passive inductor, a linear passive capacitor, and a nonlinear active memristor. Under variation of two parameters we observed a rich variety of bifurcation phenomena, including periodic, quasi-periodic, intermittent, and chaotic behaviors associated with this very simple nonlinear system. One and two-parameter bifurcation diagrams were studied. Route of period-doubling to chaos through different structures was observed. Our theoretical results are compared with the available experimental results and are found to be in good agreement with these results.

1. Introduction

Study of nonlinear dynamical systems has received considerable attention of many researches in exploring the complexity of their dynamical behaviors [1–3]. Some of these systems, under certain conditions, can display unpredictable deterministic chaos [4]. The characteristics of chaotic behavior of such systems are strongly dependent on the initial conditions and are the extreme sensitivity to the tiny changes of these conditions, the system parameters, and the broad-band frequency spectrum [3]. These can be useful and quite valuable for many practical applications [5,6], controlling, and synchronization and secure communication [7,8].

Due to the simplicity in construction and analysis, the chaotic electronic circuit is especially attractive and has several possible practical applications [9–12]. For this reason, the study of the dynamical behavior using this interesting nonlinear dynamical system was carried out in the present work.

One of the most well-known chaotic circuits, which could form an introduction to investigation of chaotic behavior, is Muthuswamy–Chua System [13], which is another form of Chua circuit [14]. This system is one of the simplest systems that are capable of producing chaos. It can exhibit a wide spectrum of dynamical phenomena, including attractors, bifurcations and routes to chaos [15–21]. There are two types of chaotic systems, autonomous and non-autonomous. Muthuswamy–Chua circuit is an autonomous system because there is no external signal injected into the system. The problem with the original version of this electronic circuit, the Chua's circuit [14], was the inductor in the circuit that had significant parasitic resistance which was not taken into account by the Chua's circuit model. As a result, the chaotic behavior was difficult to observe in this circuit. The recent versions of this dynamical system are easy to model and contain three degrees of freedom, which are necessary for observation of chaotic behavior [3]. Furthermore, the dynamical behaviors of these circuits can be described by simplest three first-order differential equations [13] capable to display a wide spectrum of dynamical phenomena.

In this paper we present a detailed bifurcation analysis of a simple electronic circuit consisting of two linear elements and one nonlinear element needed to generate all possible dynamical behaviors. Our detailed study of this electronic circuit allows to us to compare the obtained numerical results with the available experimental results.

2. Mathematical model

In the present study, we have used the same electronic circuit described in [13] and its schematic diagram is shown in Fig. 1. It is a familiar series electronic circuit consisting of three elements, a linear passive inductor (L), a linear passive capacitor (C), and a nonlinear active memristor (M) "which is a resistor with memory" [22,23].

The dynamics of this electronic circuit can be represented by set of three first-order coupled differential equations in the variables $x(t)$, $y(t)$ and $z(t)$ which give the voltage across capacitor $v_c(t)$, the current through the inductor $i_L(t)$, and the internal state of the memristive system as shown in Fig. 1. In terms of the

* Corresponding author.: Tel.: +96407801453244.
E-mail address: al_saidi_imad@yahoo.com (I.A.H.A. Al-Saidi).

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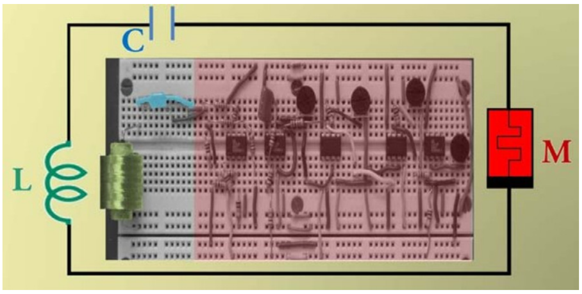

Fig. 1. Schematic diagram of the three-element memristor-based chaotic circuit. (Taken from [13]).

parameters in Fig. 1, these equations can be written in the following dimensionless form [13]:

$$\dot{x} = \frac{y}{C} \tag{1}$$

$$\dot{y} = -\frac{1}{L}\left(x + \beta\left(z^2 - 1\right)y + \frac{R_s}{I_s}y\right) \tag{2}$$

$$\dot{z} = -\frac{y}{T_s R_b C_f} - \alpha z + \frac{yz}{T_s R_a C_f} \tag{3}$$

where α and β are the system control parameters, determined by the particular values of the circuit components $C = 1$ nF a capacitance of the capacitor C, $R_s = 100\ \Omega$ is the resistance of the sensing resistor, $I_s = 10^4$ is the sensing current, and $T_s = 10^5$ is the sensing time. The other control parameters are, C_f is the capacitance, R_a and R_b are the resistances of the capacitor and the memristor, respectively.

The nonlinear dynamics of Eqs. (1)–(3) depend on the values of the circuit control parameters, corresponding to the experimental values of the parameters, and changing of these values leads to different types of dynamical behavior.

3. Results and discussion

Eqs. (1)–(3) are solved numerically using the fourth-order Runge–Kutta algorithm for different electronic circuit control parameters. The obtained results are shown in Figs. 2–5.

By varying the values of circuit parameters α and β, we construct two-parameter phase diagrams of the dynamical behavior of the electronic circuit in Fig. 1. These diagrams are drawn in the (α–β) plane for four chosen values for the inductance L (200 mH, 330 mH, 400 mH and 500 mH), as shown in Fig. 2. The values of the other circuit parameters are kept fixed as, $R_a = R_b = 1$ kΩ, $R_s = 100\ \Omega$, $C = 1$ nF, $C_f = 10$ nF, $I_s = 10^4$ and $T_s = 10^5$.

Based on our detailed numerical observations of the current $i_L(t)$ through the inductor L as a result of varying α and β, a profile of the attractor in the form of a two-parameter as shown in Fig. 2 was obtained, one can identify various interesting dynamical phenomena, including period-doubling sequence [24], quasi-periodic [25], and intermittent routes to chaos [26], as well as periodic windows within the chaotic region and reverse bifurcations. The different bifurcations are color coded in order to distinguish between them, each colored region denotes a particular type of behavior state, and these are clearly evident in the colored rectangles on the right side of the Fig. 2.

The diagrams shown in Fig. 2 were generated by the calculation of Lyapunov exponents to identify the state of the system i.e. steady state, periodic and chaotic. The periodicity of the system (period 1, period 2, etc.) was examined by drawing the Poincare

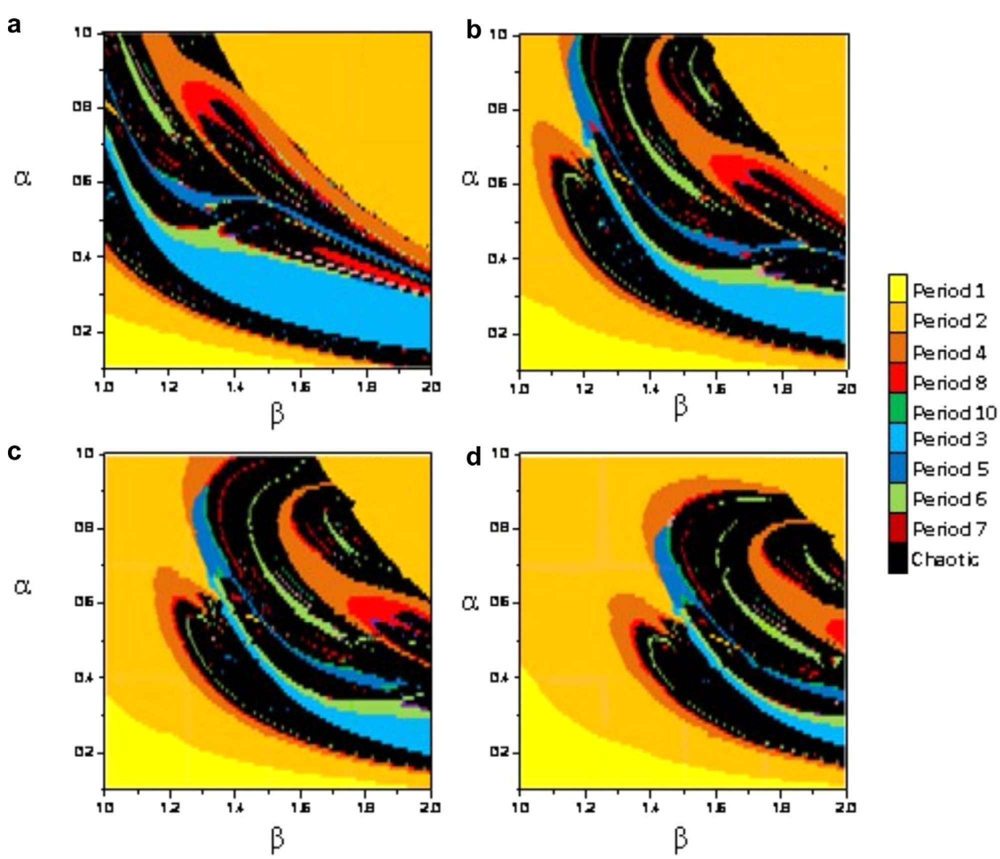

Fig. 2. Two-parameter bifurcation diagram (color map) of the dynamical behavior of the three-element memristor circuit of Fig. 1 in the (α–β) plane for (a) $L = 200$ mH, (b) $L = 330$ mH, (c) $L = 400$ mH, (d) $L = 500$ mH.