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The impact of delaying an investment decision on R&D projects in real option game

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ABSTRACT

In a research and development (R&D) investment, the cost and the project value of such an investment are usually uncertain, which thus increases its complexity. Correspondingly, the NPV (Net Present Value) rule fails to evaluate the value of this project exactly, because this method does not take into account the market uncertainty, irreversibility of investment and ability of delay entry. In this paper, we employ the real option theory to evaluate the project value of a R&D investment. Since the cost of a R&D investment is very high and the flow of the information is crowded, an investor cannot make an immediate decision every time. So, the proposed real option model is an exchange option. At the same time, combining the real option and the game theory, we can find the Nash equilibrium which is the optimal strategy. Moreover, we also study how the delayed time influences the price of the project investment and how the different delayed times effect the choice of the optimal strategies.

1. Introduction

The research and development (R&D), as a general term for activities in connection with the corporate or governmental innovations, has become a core part of the modern business world. On the one hand, the R&D investment generates new opportunities to promote economic development. On the other hand, it can further change the market structure because the achievements of R&D projects, which are always protected by governments, remove rivals from a given field. In particular, some high-tech industries, like the pharmaceutical, software and semiconductor industries, will keep an eye on the R&D investment rather than price competition. Unfortunately, the R&D is not often intended to yield immediate profits, and generally carries a greater risk and an uncertain return on investment. These characters, the market uncertainty and the ability to delay entry, are all not taken into account by the traditional net present value (NPV) method. So, the real option theory has been widely used in evaluating investment decisions, in which the managerial flexibility can be considered. In the following literature, various options evaluating projects have been extensively studied. In [1–3], and [4], it is assumed that the option exercise price and the investment cost are fixed. However, it is essential to consider the option exercise to be a stochastic variable. To this purpose, in [5] a European exchange option is used to value the assets distribute dividends. Further, in [6] a model is developed to value an American exchange option through approximating American puts, and a model is presented to evaluate a European compound exchange option. Moreover, in [7] the exchange option is also employed to value R&D investments. In the above models, the assets distribute "dividends" are considered. If a R&D investment project is postponed, it can produce the opportunity costs in [1]. Because of the managerial flexibility of the R&D investment, we want to give corporations a right to choose again at the delayed time besides the initial time when they make the R&D investments. So, we choose the exchange option to evaluate R&D projects.

During recent years, it has been recognized that in a dynamic environment, the combination of real option theory and the game theory becomes an important tool to solve the problems of valuing R&D projects. In this field, many researchers contribute a lot of works. The first literature combining the real options theory with the game theory is written for a duopoly market with a continuous-time model under the product market competition [8], and the paper [9] provides the first rigorous derivation of a Nash-equilibrium in a real options framework. Also, we use the Nash equilibrium solutions as the best strategies. A real option game model is established in [10] to study an investment on a winner-takes-all patent system with irreversible investment costs and uncertain revenues, which may shed light on strategic delay in patent

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races and explain the role of first vs second movers. Furthermore, [11] and [12] consider innovation with uncertainty over completion and time delays, which can explain some phenomena like the faster exit and the delayed commercialization. In addition, [13] derives a fully dynamic model of investment under uncertainty for first-mover advantage contexts while the incomplete information is assumed and the competition is considered.

In this paper, we analyze two corporations in a dynamic market, and consider their interactions with externality. Here we assume that the “information revelation” is generated by the R&D investments about their success or failure which has been investigated in [14,15], and [16]. This means that the new information can be revealed, which might affect the follower’s probability of the R&D investments by delaying an investment decision when a corporation invests at the initial time whether it is successful or not. The mutual information generates the positive network externality, which is studied in some papers. For example, [16] proposes an option game between two corporations who invest in R&D with externality, and studies the corporations’ optimal strategies. Moreover, [17] considers a greater capability to take advantage of future growth opportunities, which leads to the capture of a greater share of the market. In [18], several real options models are supplied for several different economic contexts with constant or non-constant investment costs, and [19] refines the definition of externality and considers a cooperation between two corporations. However, in the previous literature, the effect of the delayed time on the Nash equilibrium solutions and the sets of optimal strategies is not considered.

In this work, the delayed time, which is the second chance for corporations to choose whether they invest or not if they do not invest at the initial time, plays a crucial role. The delayed time is influenced by the corporations’ operating cash flows deeply. Without considering the delayed time as an endogenous variable, we ignore the impact on the R&D investment by the corporations’ operating cash flows and cannot find the change of optimal strategy under different delayed times. In addition, the delayed time will affect the value of an exchange option and will further influence the Nash equilibrium solutions which are defined as the best strategies of corporations. To this end, in the present paper, we will study the above mentioned impacts of the delayed times.

This paper is organized as follows. In Section 2, a simple and compound European exchange option pricing model and a basic model game are reviewed. In Section 3, we will introduce the critical market values in order to determine the Nash equilibrium solutions. Section 4 is concerned with the sets of optimal strategies under different delayed times. Finally, in Section 5 we have concluding remarks.

2. Exchange options methodology and the basic model game

Real options theory has emerged since the eighties as a valuation technique, and especially this theory is appropriate for evaluating the investments with high uncertainty, since it can capture the value of managerial flexibility to adapt decisions in response to unexpected market developments. In addition, it is easily used to model the competitive investment decisions by mixing the concepts from other theories.

It is well known that game theory is the study of mathematical models of conflict and cooperation among decision makers, and it has been paid a great deal of attention in the academic field over the last decades, such as economics, biology, mathematics, and political science. In this paper, we combine real options theory with game theory to study the R&D investments of two firms, in which how the delayed time influences the project investment and how the different delayed times effect the choice of the optimal strategies are addressed.

2.1. Exchange options methodology

In this section, we review a basic model to value European exchange options. Following [5], the valuation of a simple European exchange option (SEEO), which is a right to exchange asset C to asset V at time T, can be obtained. We assume that asset V and asset C follow the following geometric Brownian motions given by

$$\frac{dV}{V} = (\mu_v - \delta_v)dt + \sigma_v dB_v,$$

$$\frac{dC}{C} = (\mu_c - \delta_c)dt + \sigma_c dB_c,$$

with

$$\mathrm{cov}\left(\frac{dV}{V}, \frac{dC}{C}\right) = \rho_{vc}\sigma_v\sigma_c dt,$$

where μ_v and μ_c are the equilibrium expected rates of returns on the two assets, δ_v and δ_c are the corresponding “dividend-yields”, σ_v and σ_c are the respective volatilities, B_v and B_c are two standard Brownian motions and have a correlation coefficient ρ_{vc}. Here the coefficients μ_v, μ_c, δ_v, δ_c, σ_v, σ_c, and ρ_{vc} are assumed to be nonnegative constants. The valuation of a SEEO on dividend-paying assets at $t = 0$ is given by [5]

$$s(V, C, T) = Ve^{-\delta_v T}N(d_1(P, T)) - Ce^{-\delta_c T}N(d_2(P, T)), \tag{2.1}$$

where

- V and C are the gross project value and investment cost, respectively;
- $P = \frac{V}{C}$, $\sigma = \sqrt{\sigma_v^2 - 2\rho_{vc}\sigma_v\sigma_c + \sigma_c^2}$, and $\delta = \delta_v - \delta_c$;
- $d_1(P, T) = \dfrac{\log P + \left(\frac{\sigma^2}{2} - \delta\right)T}{\sigma\sqrt{T}}$, and $d_2 = d_1 - \sigma\sqrt{T}$;
- $N(d)$ is the cumulative standard normal distribution.

According to [6], we develop a model to value the compound European exchange option (CEEO) whose final payoff at the maturity date t_1 is $c(s, \varphi C, t_1) = \max(0, s - \varphi C)$. The CEEO valuation at $t = 0$ is given by

$$\begin{aligned} c(s(V, C, T), \varphi C, t_1) = {} & Ve^{-\delta_v T}N_2\left(d_1\left(\frac{P}{P^*}, t_1\right), d_1(P, T); \rho\right) \\ & - Ce^{-\delta_c T}N_2\left(d_2\left(\frac{P}{P^*}, t_1\right), d_2(P, T); \rho\right) \\ & - \varphi Ce^{-\delta_c t_1}N\left(d_2\left(\frac{P}{P^*}, t_1\right)\right), \end{aligned} \tag{2.2}$$

where

- φ is the exchange ratio of the CEEO;
- t_1 is the expiration date of the CEEO;
- T is the expiration date of the SEEO, where $T > t_1$;
- $\tau = T - t_1$ is the time to the maturity of the SEEO and $\rho = \sqrt{\frac{t_1}{T}}$;
- $d_1(\frac{P}{P^*}, t_1) = \frac{\log(\frac{P}{P^*}) + (-\delta + \frac{\sigma^2}{2})t_1}{\sigma\sqrt{t_1}}$, and $d_2(\frac{P}{P^*}, t_1) = d_1(\frac{P}{P^*}, t_1) - \sigma\sqrt{t_1}$;
- P^* is the critical price ratio that solves the following equation:

$$P^* e^{-\delta_v \tau}N(d_1(P^*, \tau)) - e^{-\delta_c \tau}N(d_2(P^*, \tau)) = \varphi; \tag{2.3}$$

- $N_2(a, b; \rho)$ is the standard bivariate normal distribution function evaluated at a and b with correlation ρ.