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Delay-range-dependent synchronization of drive and response systems under input delay and saturation

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ABSTRACT

This paper addresses the synchronization of nonlinear drive and response systems under input saturation and subject to input time-delay. In considering generalized forms of the systems, their dynamics are assumed to satisfy the one-sided Lipschitz condition along with the quadratic inner-boundedness rather than the conventional Lipschitz condition. Further, the time-delays are handled by application of the delay-range-dependent methodology, rather than the delay-dependent one, utilizable for both short and long time-delays. Synchronization controller designs are provided by application of the Lyapunov–Krasovskii functional, local sector condition, generalized Lipschitz continuity, quadratic inner-boundedness criterion and Jensen's inequality. To the best of the authors' knowledge, a delay-range-dependent synchronization control approach for the one-sided Lipscitz nonlinear systems under input delay and saturation constraints is studied for the first time. A convex-routine-based solution to the controller gain formulation by application of recursive nonlinear optimization using cone complementary linearization is also provided. The proposed methodology is validated for synchronization of modified Chua's circuits under disturbances by considering the input delay and saturation constraints.

1. Introduction

Synchronization of complex nonlinear systems, made possible by means of a feedback controller, has vast applications in robotics, secure communications, image processing, avionics, information processing, and biomedical networks [1–5]. The main purpose of synchronization control is to establish a coherent behavior between the drive and response systems by applying a feedback of the difference between the states or outputs [6–8]. Different control schemes and tools for synchronization of nonlinear systems have been realized: Nonetheless, selection of a synchronization controller and type of control methodology depend on the circumstances and actual environment, which often vary from case to case. For instance, adaptive controllers are used for adaptation of wide-ranging unknown parameters, as seen in [9]. Robust controllers, meanwhile, are applicable to fast-varying changes and perturbations, as demonstrated in [10]. Constrained controllers are employed to deal with input, state or output restraints such as saturation. Likewise, consensus controllers are designed to deal with specific communication and network protocols (see [11,12]). Output and state feedback controllers are employed according to the availability of states and outputs. Observer-based controllers are preferable to attain the advantages of the state feedback approaches when the states are unknown [13]. Disturbance-observer-based controllers are utilized for adaptive cancellation of unknown matching disturbances [14]. Controller design for effectual synchronization remedy of nonlinear systems is still a thought-provoking research area, especially in view of system dynamics, uncertainties, various constraints, and overall performance goals.

Controllers for synchronization of nonlinear time-delay systems are designed to utilize time-delay data such as lower and upper bounds, the rate of delay, and the number of delays appearing in the state, input or output. Conventional controllers for synchronization of nonlinear systems might not guarantee synchronization, because time-delays can cause oscillations and instability in the response of the synchronization error. Several attempts to synthesize synchronization controllers for time-delay systems have been made, exclusively by employing delay-independent and delay-dependent methods and by applying elusive delay-range-dependent techniques. For instance, two delay-dependent synchronization control methods for Lur'e systems based on delayed

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feedback via the partitioning-interval approach were developed in [15]. Zhang et al. [16] utilized range-of-delay information to develop a global synchronization methodology for complex networks under stochastic disturbances. Fei and coauthors [17] followed the delay-partitioning approach in studying the coherent behavior of a complex network with interval time-varying delay coupling. Li et al. [18] utilized a novel Lyapunov function in their evaluation of a delay-range-dependent synchronization control mechanism for Lur'e systems. The work in [19] achieved the synchronization of chaotic systems with time-varying state delays and delayed nonlinear coupling between the drive and response systems. Recently, Ma and Jing [20] developed, by means of a local sector condition, a delay-independent state-feedback control approach for synchronization of uncertain nonlinear systems with time-varying state delays and input saturation. More recently, Cai and coworkers [21] have reported delay-dependent synchronization conditions of singularly perturbed systems with coupling delays.

Works on the delay-range-dependent stability investigation, control and synchronization proficiencies (owing to their utilities for dealing with short as well as long time-delays in the state, output, input or coupling between nonlinear systems) are proceeding apace. The literature on synchronization of the nonlinear systems using a delay-range-dependent approach by incorporating the input saturation nonlinearity and time-delays, however, is deficient. There are two major issues with the existing synchronization techniques for the nonlinear time-delay systems. First, most of the above-mentioned studies employ a conservative continuity condition like the conventional Lipschitz condition for the derivation of the synchronization control strategies. The literature of mathematics has developed a less conservative one-sided Lipschitz condition, which can be used to represent the Lipschitz nonlinear systems as a specific case of the one-sided Lipschitz nonlinear systems. Moreover, the one-sided Lipschitz constant may have a smaller value than the Lipschitz constant, which fact can be more effectively applied for derivation of the controllers to synchronize nonlinear oscillators with large or region dependent Lipschitz constants. Second, the input saturation nonlinearity cannot be ignored in practical systems because an untreated saturation nonlinearity can lead to oscillations, lags, overshoots, undershoots, performance abasement, and divergence of the closed-loop system response. For synchronization of the nonlinear systems under input time-delays, dealing with the saturation consequences is a non-trivial research dilemma owing to simultaneous considerations of the input saturation and the input delay effects.

This paper introduces controller design for synchronization of nonlinear systems under input saturation and subject to input time-delay varying within an interval of known lower and upper bounds. By utilizing the Lyapunov–Krasovskii (LK) functional, one-sided Lipschitz condition, quadratic inner-boundedness, the range of the input delay, the limit on the derivative of the delay, the local sector condition for input saturation and Jensen's inequality, nonlinear matrix inequalities are derived to determine an appropriate controller gain matrix, specifically by providing an estimate of the region of stability in terms of synchronization error. From these principal design conditions, novel synchronization controller design conditions for Lipschitz nonlinear systems, both for the delay-dependent case and for the scenario of an unknown bound on the delay-rate, are derived. Moreover, the proposed method is extended for robust synchronization of nonlinear systems under input lag and saturation by considering the L_2 norm-bounded perturbations in evaluating the allowable bound of the disturbance and disturbance attenuation level at the state estimation error.

The main contributions of the paper are summarized as follows: (i) To the best of our knowledge, delay-range-dependent synchronization of the nonlinear systems under input saturation and input delay, to deal with the practical limitations of actuators, is addressed for the first time. (ii) An inaugural treatment of synchronization of time-delay in one-sided Lipschitz nonlinear systems is provided. Such an approach is less conservative and can be employed to synchronize a broader class of nonlinear systems than the conventional Lipschitz systems. (iii) An estimate of the region of stability in terms of the difference between initial conditions of the nonlinear master-slave systems under input delay and saturation is provided. (iv) A robust synchronization method for time-delay nonlinear systems with one-sided Lipschitz nonlinearities, input delay, input saturation, and external perturbations is explored. In this regard, an upper bound on the L_2 norm of the synchronization error in terms of the initial condition and disturbances and the region in which the synchronization error remains bounded are revealed.

Additionally, a numerically tractable approach is outlined for determining the synchronization controller gain matrix, parameters representing the ellipsoidal region of stability, and scalars to constitute bounds on the synchronization error by utilizing the cone complementary linearization algorithm. Finally, a numerical simulation example is provided to demonstrate the effectiveness of the proposed methodology for synchronization of input-constrained modified chaotic Chua's circuits in the presence of input delays and disturbances.

This paper is organized as follows: the drive and response systems are described in Section 2. In Section 3, various synchronization controller designs for dealing with nonlinearities, delays, input saturation and disturbances are introduced. In Section 4, simulation results are provided. Concluding remarks are rendered in Section 5.

Standard notation is used in this paper. A block diagonal matrix is denoted as $\mathrm{diag}(x_1, x_2, \ldots, x_m)$, where $x_1, x_2, \ldots, x_m$ are entries at the corresponding diagonal blocks. L_2 norm for a vector $x \in R^n$ is represented as $\|x\|_2$ and the ith row of a matrix A is assigned as $A_{(i)}$. $\langle w, v\rangle$ represents the inner product between two vectors w and v of matching dimensions. The saturation nonlinearity is defined by $\Psi_{(i)}(u_{(i)}) = \mathrm{sgn}(u_{(i)})\min(\bar{u}_{(i)}, |u_{(i)}|)$ for the saturation bound given as $\bar{u}_{(i)} > 0$. Positive definite and positive semi-definite matrices are represented as $Y > 0$ and $Y \geq 0$, respectively, for a symmetric matrix Y.

2. System description

Consider a master (or drive) system

$$\frac{dx_m}{dt} = Ax_m + f(t, x_m) + d_1,$$
$$y_m(t) = Cx_m, \tag{1}$$

where $x_m \in R^n$, $y_m \in R^p$ and $d_1 \in R^m$ represent the state, output and disturbance vectors, respectively. A and C are constant matrices of appropriate dimensions, and $f(t, x_m) \in R^n$ denotes the nonlinear dynamics in the system. The slave (or response) system is given by

$$\frac{dx_s}{dt} = Ax_s + f(t, x_s) + B\Psi(u(t-\tau)) + d_2,$$
$$y_s(t) = Cx_s, \tag{2}$$

where $x_s \in R^n$, $y_s \in R^p$, $u \in R^q$ and $d_2 \in R^m$ are the state, output, control input and disturbance to the response system, respectively, B is the input matrix, and $\Psi(u)$ is the input saturation vector-function. The input time-delay satisfies

$$0 \leq \tau_1 \leq \tau(t) \leq \tau_2, \tag{3}$$

$$\dot{\tau}(t) \leq \mu. \tag{4}$$

Assumption 1. The function $f(t, x_m)$ satisfies the one-sided Lipschitz condition given as

$$\langle f(t, x_m) - f(t, x_s), x_m - x_s\rangle \leq \rho\|x_m - x_s\|^2 \tag{5}$$