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Stability switches and bifurcation in a two-degrees-of-freedom nonlinear quarter-car with small time-delayed feedback control

G.I. Koumene Taffo, M. Siewe Siewe*, C. Tchawoua

Laboratory of Mechanics, Materials Science and Structures, Research and Training Unit for Doctorate in Physics and Applications, Postgraduate School of Science, Technology and Geosciences, University of Yaounde I, P.O. Box 812, Yaounde, Cameroon

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ABSTRACT

In this paper, we investigate a two-degrees-of-freedom nonlinear quarter-car model with time-delayed feedback control. It is well known that a time delay has destabilizing effects in mathematical models. However, delays are not necessarily destabilizing. In this work we explore a system where a time delay can be both stabilizing and destabilizing. Using the generalized Sturm criterion, the critical control gain for the delay-independent stability region and critical time delays for stability switches are derived. It is shown that there is a small parameter region for delay-independently stability of the system. Once the controlled system with time delay is not delay-independently stable, the system may undergo stability switches with the variation of the time delay. These stability switches correspond to Hopf bifurcations that occur when the time delays cross critical values. Properties of Hopf bifurcation such as direction and stability of bifurcating periodic solutions are determined by using the normal form theory and centre manifold theorem. Numerical simulations are provided to support the theoretical analysis. The critical conditions can provide a theoretical guidance for the design of vehicles with significant reduction of vibration in order to increase passengers ride comfort.

1. Introduction

For many years, there has been an increasing interest in the study of the optimal control and the design of vehicle suspension to reduce vibration and improve the stability of the vehicle. The suspensions' systems include the passive suspension [1], the active suspension [2,3] and the semi-active suspension [4]. Compared with the conventional passive suspension of vehicle model, the active suspension can improve suspension performances, such as ride comfort and road holding. Recently many new applications of active and semi-active control procedures and special devices to minimize vehicle vibrations have been developed [5]. In this field, the suppression of vibration by active feedback control has received particular attention.

In practice, unavoidable time-delays frequently appear in the controlled mechanical or structural systems, especially in hydraulic actuators used in active suspensions of ground vehicles and active tendons of tall buildings. It has been found that time-delay not only retards the systems, but often it is a source for birth of a limit cycle, loss of stability and occurrence of bifurcations and chaos. However, an appropriate time-delayed feedback control law can improve system performance or even stabilize an unstable system [6]. Therefore we need to deepen our understanding of the complexity induced by time-delays. Shayer and Campbell [7] and Liao and Chen [8] studied the Hopf bifurcation and double-Hopf bifurcation respectively in DDEs with multiple time-delay feedback. Xu and Yu [9] investigated delay-induced bifurcation in a non-autonomous system with time-delayed velocity feedback. Raghavendra and Pravin [10] studied resonance, stability and chaotic vibration of a quarter-car vehicle model with time-delay feedback. They also studied the stability of a nonlinear 2 degrees-of-freedom vehicle system with multiple time-delays [11]. Koumene Taffo and Siewe Siewe [12] investigated parametric resonance, stability and heteroclinic bifurcation in a nonlinear oscillator with time-delay.

Some studies have already shown the importance of solving the time-delay in control systems [13,30,34]. When a vehicle runs across a bump, one may feel violent jerk. The time span of the vehicle running across the bump is so short that it is necessary to consider the adverse effect of time-delay on control performance. Time-delayed feedback is considered to be an ideal choice to suppress the nonlinear responses of dynamical systems, and time-delayed controllers have been applied to plenty of linear and nonlinear dynamical systems. It is well known that a controlled system

* Corresponding author.
E-mail addresses: kgildasigor@yahoo.fr (G.I. Koumene Taffo), martinsiewesiewe@yahoo.fr (M. Siewe Siewe), ctchawa@yahoo.fr (C. Tchawoua).

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is asymptotically stable if all the roots of the corresponding characteristic equation have negative real parts.

For a linear time-invariant system with time delays, however, the characteristic equation becomes transcendental due to the exponential functions associated with time delays. The transcendentality brings an infinite number of characteristic roots, which are cumbersome to handle [14]. To resolve this difficulty, a number of methodologies have been suggested that assess the stability of time delay systems, such as Stépán–Hassard method [15], D-subdivision method [16], and its modified version [17]. In Wang and Hu [18], the generalized Sturm criterion was used to determine whether a polynomial equation has positive real roots. Wang et al. [19] studied delay-independent stability of retarded dynamic systems of multiple degrees of freedom. Some studies have shown that there are stability switches with variations in time delay in time-delayed feedback systems. Zhao [20] investigated stability of a two-dimensional airfoil with time-delayed feedback control. Li et al. [21] studied the stability and chaotification of vibration isolation floating raft systems with time-delayed feedback control. Xu et al. [22] worked on stability and Hopf bifurcation of a two-dimensional supersonic airfoil with a time-delayed feedback control surface. From these studies, it was found that with proper design, the time delay could be a more efficient way to control instability than the conventional control strategies without time delay.

Hopf bifurcation is a very interesting topic when it comes to the study of nonlinear delayed systems. This study consist of investigating the existence of the Hopf bifurcation as well as determining the stability and direction of the bifurcation. The normal form theory and centre manifold theorem approach developed by Hassard et al. [23] is widely used in the investigation of Hopf bifurcations in delayed nonlinear systems [24–29]. Recently, Xu et al. [22] studied the stability and Hopf bifurcation of a two-dimensional supersonic airfoil with a time-delayed feedback control surface and it was pointed out that, the nonlinear aeroelastic system undergoes a sequence of Hopf bifurcations if the time delay passes the critical values and at the vicinity of the Hopf bifurcation there exist bifurcating periodic solutions.

The main objective of the present study is to apply a systematic methodology leading to optimum combinations of the suspension damping and feedback gain of a ground vehicle subjected to road excitation. Most of the previous studies on the subject have dealt with car models possessing linear characteristics or single-degree-of-freedom mechanical models [10]. Recently great attention had been paid to the study of two-degrees-of-freedom models, Litak and Borowiec [31] studied a 2-DoF quarter-car and revealed the transition to chaos, by applying Melkinov theory and recurrence approach. Donescu et al. [32] examined chaotic behaviour of a nonlinear oscillator with kinematic excitation. However, in the case of two-degrees-of-freedom models, little attention has been paid to revealing and studying important effects related to increase in time delay and feedback gain [11].

In the present study, the stability problem of a two-degrees-of-freedom nonlinear quarter-car with time-delayed feedback control is investigated. The paper is organized as follows, in Section 2 we present the mathematical model of the two-degrees-of-freedom nonlinear quarter-car with time-delayed feedback control. In Section 3, the stability analysis of the linearized quarter-car model is conducted by the generalized Sturm criterion. The delay-independent stability, critical time delays and Hopf bifurcations of the quarter-car model are investigated. Section 4 is devoted to the study of properties of Hopf bifurcation such as direction and stability using the normal form theory and centre manifold theorem. General bifurcation diagrams are also plotted to have an overview of the nonlinear behaviour of the system in the presence of external excitation due to road profile. We then end in Section 5 with a general conclusion.

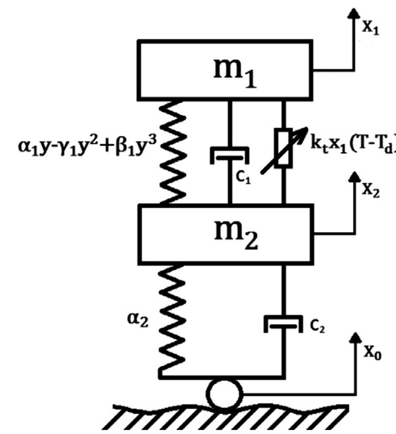

Fig. 1. Two-degrees-of-freedom nonlinear quarter-car model.

2. Mathematical model of the nonlinear quarter-car model

The model under investigation is presented in Fig. 1. The model consists of two rigid bodies, the unsprung mass m_2 representing the axle and wheel of the vehicle while the sprung mass m_1 represents the portion of the vehicle body located over the axle. The model has the degrees of freedom X_1 and X_2, corresponding to the vertical motions of the sprung and unsprung mass respectively. The road profile is represented by the displacement function $X_0 = a\cos\omega t$ which describes the harmonic corrugation of a road. The springs and dampers represent the suspension and tyre stiffness and damping, respectively. The vehicle body is supported by a linear damper and a nonlinear spring which possesses quadric and cubic nonlinearity. The unsprung mass is then linked to the road using a linear damper and a linear spring which characterized the compressibility of the pneumatic tyre. A time-delayed position feedback is introduced between the vehicle body and the unsprung mass to implement time-delayed feedback control.

The equations of motion of the nonlinear quarter-car model without time-delayed feedback control are given by

$$\begin{aligned} &m_1\ddot{X}_1 + c_1(\dot{X}_1 - \dot{X}_2) + \alpha_1(X_1 - X_2) \\ &\quad - \gamma_1(X_1 - X_2)^2 + \beta_1(X_1 - X_2)^3 = 0, \\ &m_2\ddot{X}_2 + c_2\left(\dot{X}_2 - \dot{X}_0\right) - c_1(\dot{X}_1 - \dot{X}_2) + \alpha_2(X_2 - X_0) \\ &\quad - \alpha_1(X_1 - X_2) + \gamma_1(X_1 - X_2)^2 - \beta_1(X_1 - X_2)^3 = 0, \end{aligned} \tag{1}$$

where c_1 is the damping coefficient of the nonlinear vibration isolator; α_1, γ_1 and β_1 are the linear, quadric and cubic stiffness coefficients of the nonlinear vibration isolator, respectively. The relation between force and displacement for the nonlinear spring is $f = \alpha_1\delta - \gamma_1\delta^2 + \beta_1\delta^3$. c_2 and α_2 are the damping and stiffness coefficients of the tyre, respectively.

The governing equations with time-delayed feedback control are given by

$$\begin{aligned} &m_1\ddot{X}_1 + c_1(\dot{X}_1 - \dot{X}_2) + \alpha_1(X_1 - X_2) \\ &\quad - \gamma_1(X_1 - X_2)^2 + \beta_1(X_1 - X_2)^3 = k_t X_1(T - T_d), \\ &m_2\ddot{X}_2 + c_2\left(\dot{X}_2 - \dot{X}_0\right) - c_1(\dot{X}_1 - \dot{X}_2) \\ &\quad + \alpha_2(X_2 - X_0) - \alpha_1(X_1 - X_2) + \gamma_1(X_1 - X_2)^2 \\ &\quad - \beta_1(X_1 - X_2)^3 + k_t X_1(T - T_d) = 0, \end{aligned} \tag{2}$$

where k_t is the feedback gain and $T_d \geq 0$ is the time delay of the feedback control. Substituting $X_0 = a\cos\omega t$ in Eq. (2), its dimensionless form is written as

$$\begin{aligned} &\ddot{x}_1 + C_1A_1(\dot{x}_1 - \dot{x}_2) + (x_1 - x_2) - \xi_1(x_1 - x_2)^2 \\ &\quad + A_3(x_1 - x_2)^3 = k_f x_1(t - \tau), \\ &\ddot{x}_2 + C_2MA_1\dot{x}_2 - C_1MA_1(\dot{x}_1 - \dot{x}_2) + MA_2x_2 \end{aligned}$$