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Nonlinear modelling of the interaction between phytoplankton and zooplankton with the impulsive feedback control☆

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ABSTRACT

In this paper, a mathematical model including the phytoplankton and zooplankton with the impulsive feedback control is presented. The sufficient conditions for the existence of the order-1 and order-2 periodic solutions are obtained by using the geometrical theory of semi-continuous dynamic system. The stability of the order-1 periodic solution is discussed by the analogue of the Poincaré criterion. Finally, our results are justified by the numerical simulations.

1. Introduction

Plankton are usually classified into phytoplankton and zooplankton and they play a vital role in the circulation of matter and energy in the lake ecosystem. Phytoplankton are primary producers and occupy the first trophic level in the food chain [1]. Phytoplankton can make an important contribution to the climate regulation by absorbing the carbon dioxide from the atmosphere and releasing a huge amount of oxygen through the photo-synthesis in presence of sunlight. Phytoplankton are usually regarded as most favorable food sources for fish and other aquatic animals [2]. A remarkable feature associated with phytoplankton is the occurrence of rapid and massive bloom formation when too many nutrients(phosphorus and nitrogen) loading from human activities enter rivers and lakes, which is also known as eutrophication [3]. There are many ways to decrease phytoplankton biomass. It involves chemical method (adding chemical reagents to the water) and biological method (releasing zooplankton or competitor) [4]. Chemical method is useful because they quickly remove a significant portion of phytoplankton and sometimes provide the only feasible method for preventing the phytoplankton blooms, while chemical method may produce the secondary pollution. Biological method can reduce pollution and protect the ecological balance, with little cost to minimal effect on the environment.

Many researchers are investigating to combine the biological and chemical methods so that the effects on the phytoplankton and the natural enemies are considered and are sustainable in the long term. It has been proving that combination of the different control strategy is more effective management of phytoplankton [5], which implies different control strategies should work together rather than against each other.

Many researchers investigated the dynamic mechanisms of the eutrophication by means of formulating the mathematical model to decrease the negative effect of the eutrophication on the lake ecosystem [6–14]. Banerjee et al. [8] proposed a toxic-phytoplankton-phytoplankton model and obtained the stability of biologically feasible equilibria in terms of reproduction numbers zooplankton system. In [9], authors formulated a seasonally and periodically forced phytoplankton-zooplankton model and obtained a variety of complex population dynamics including chaos, quasi-periodicity, and periodic resonance. In [13], a Nitrogen-Phosphorus-Phytoplankton model is proposed in a water ecosystem and the existence and stability of equilibria are proved by constructing the Lyapunov function. Considering the phytoplankton growth influenced by many factors, Patrick and Fox [14] reduced the amounts of the phytoplankton below the

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threshold of the eutrophication occurrence by means of the biomanipulation.

In fact, impulsive state feedback control has been widely applied in many regions [15–19]. Wei [15] et al. investigated a homoclinic bifurcations of a prey-predator system with impulsive state feedback control and they obtained the order-1 homoclinic cycle on the basis of the theory of rotated vector fields. In [18], authors formulated a chemostat model with the impulsive state feedback control and obtained the existence and stability of the order-1 periodic solution by using successor function and Poincaré map. Authors [19] investigated a nonlinear modelling of ethanol inhibition with the state feedback control and they proved the sufficient conditions for existence of the period-1 solution and period-2 solutions are obtained based on the theory of the impulsive semi-dynamical system. However, little is known about the application of impulsive state dependent feedback control to plankton model. In this paper, we will formulate a nonlinear modelling of the interaction between phytoplankton and zooplankton with the impulsive feedback control.

The paper is organized as follows: a nonlinear modelling of the interaction between phytoplankton and zooplankton with the impulsive feedback control is proposed in Section 2. The existence and stability of order-1 periodic solution are investigated in Section 3. In Section 4, the existence of order-k periodic solution ($k \geq 2$) is investigated. Some numerical simulations are presented in Section 5. Finally, we give a brief discussion.

2. Development of model and preliminaries

Since a remarkable feature associated with phytoplankton is the occurrence of rapid and massive bloom formation when too many nutrients(phosphorus and nitrogen) loading from human activities enter rivers and lakes, it would be useful to foresee approaching a critical threshold value(which can be monitored by experiment) for eutrophication in lake ecosystem so that actions to prevent eutrophication could be taken [20]. The critical threshold value is usually regarded as the lowest phytoplankton concentration that will cause eutrophication [21]. The control actions must be taken once the phytoplankton concentration reaches the critical threshold value. Motivated by the references [6–21], we will give some assumptions to formulate a nonlinear modelling of the interaction between phytoplankton and zooplankton with the impulsive feedback control.

(a) We suppose the phytoplankton and zooplankton are uniformly distributed in a given lake ecosystem; $P(t)$ and $Z(t)$ denote the concentrations of the toxin-producing phytoplankton and zooplankton at time t, respectively.
(b) Assume phytoplankton population $P(t)$ can grow exponentially before reaching the critical threshold value as the threshold level is relatively small compared with the carrying capacity of the phytoplankton.
(c) Suppose α is the uptake rate of zooplankton on the toxin-producing phytoplankton. β denotes the ratio of biomass conversion. μ is the death rate of zooplankton.
(d) Assume the distribution of toxic substance is denotes as $\frac{\delta P}{\gamma+P}, (\delta > 0, \gamma > 0)$, which ultimately contributes to the death of zooplankton population, where γ is a half-saturation constant, δ denotes the rate of toxin liberation by toxin-producing phytoplankton.
In this paper, the main purpose is to control the phytoplankton concentration lower than a critical threshold value(the predetermined value) once the phytoplankton concentration reaches the critical threshold value of the algal bloom outbreaks, that is, we will release the natural enemies and spray the chemical reagents when the phytoplankton concentration reaches the critical threshold value.
(e) Let h show the predetermined threshold value of the algal bloom outbreaks, which can be obtained by using a monitoring device and it is a lowest phytoplankton concentration causing the algal bloom outbreaks. τ shows the amount of the zooplankton released at the time t. We suppose the chemical reagents only harm phytoplankton and $E(0 < E < 1$, which can be obtained by means of experiments) denotes the proportion of phytoplankton reduction by spraying chemical reagents.

From assumptions $(a)-(e)$, we can formulate the following model with the impulsive state feedback control:

$$\begin{cases} \left.\begin{aligned} \frac{dP}{dt} &= (r-\alpha Z)P, \\ \frac{dZ}{dt} &= (\beta P - \mu - \frac{\delta P}{\gamma + P})Z, \end{aligned}\right\} & P < h, \\ \left.\begin{aligned} \Delta P &= P(T^+) - P(T^-) = -EP, \\ \Delta Z &= Z(T^+) - Z(T^-) = \tau, \end{aligned}\right\} & P = h, \\ P(0^+) = P_0 \geq 0, \\ Z(0^+) = Z_0 \geq 0, \end{cases} \tag{2.1}$$

where P_0 and Z_0 denote the initial concentrations of phytoplankton and zooplankton population, $\Delta P = P(t^+) - P(t), \Delta Z(t) = Z(t^+) - Z(t)$. We assume the initial concentration of the phytoplankton population is always less than the critical threshold value h, that is, $P_0 < h$, $Z_0 > 0$.

Obviously, system (2.1) includes three parts: ordinary differential equations, control resetting and initial conditions. The first two ordinary differential equations of system (2.1) show it is not necessary to take any control measure if the phytoplankton concentration in the given lake ecosystem is lower than the critical threshold value h. The control resetting of system (2.1) implies the phytoplankton concentration reaches the critical threshold value which may be detected by the monitoring devices, the control measures should be taken to decrease the phytoplankton concentration lower than the critical threshold value h.

Before discussing the periodic solution of system (2.1), we first give some definitions and lemmas.

Definition 2.1. [22] The function $d(s) = f(s) - s$ or $s = f(s)$ is called a successor function.

Definition 2.2. The function $LambertW(x)$ is defined to be multi-valued inverse of the function $x \to xe^x$ satisfying

$$LambertW(x)\exp(LambertW(x)) = x.$$

Hence if $x > -1$, then the function xe^x has the positive derivative $(x+1)e^x$. $LambertW(0, x)$ is the inverse function of xe^x on the interval $[-1, \infty)$. For the nature of this study, both $W(0, x)$ and $W(-1, x)$ will be employed only for $x \in [-\exp(-1), 0])$ because both functions are real values for x in this interval. For more details of the $LambertW$ function in [23].

Lemma 2.3. *[24] The T-periodic solution $S(t) = \xi(t), x(t) = \eta(t)$ of the system*

$$\begin{cases} \left.\begin{aligned} \frac{dS}{dt} &= P(S,x), \\ \frac{dx}{dt} &= Q(S,x), \end{aligned}\right\} & \phi(S,x) \neq 0, \\ \left.\begin{aligned} \Delta S &= \alpha(S,x), \\ \Delta x &= \beta(S,x), \end{aligned}\right\} & \phi(S,x) = 0, \end{cases} \tag{2.2}$$

is orbitally asymptotically stable if the Floquet multiplier μ_2 satisfies the condition $|\mu_2| < 1$, where

$$\mu_2 = \Pi_{k=1}^{q} \Delta_k \exp\left[\int_0^T (\frac{\partial P}{\partial S}(\xi(t), \eta(t)) + \frac{\partial Q}{\partial x}(\xi(t), \eta(t)) dt\right],$$