Chaos, Solitons and Fractals 87 (2016) 268–275

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Mixed multifractal analysis of China and US stock index series

Meifeng Dai[a,*], Jie Hou[a], Jianyu Gao[b], Weiyi Su[c], Lifeng Xi[d], Dandan Ye[a]

[a] Nonlinear Scientific Research Center, Faculty of Science, Jiangsu University, Zhenjiang, Jiangsu, 212013, P.R. China
[b] Sales Department, Zhenjiang Branch, Bank of China, 212001, Zhenjiang, Jiangsu, P.R. China
[c] Department of Mathematics, Nanjing University, Nanjing, 210093, P.R. China
[d] Institute of Mathematics, Zhejiang Wanli University, Ningbo 315100, P.R. China

ARTICLE INFO

Article history:
Received 11 September 2015
Revised 24 March 2016
Accepted 13 April 2016
Available online 25 April 2016

Keywords:
Multifractal analysis
Mixed multifractal spectrum
Stock market

ABSTRACT

In this paper, we study mixed multifractal properties of stock index series both in China i.e., SSEC, SZSE and US i,e., DJIA, NASDAQ by mixed multifractal analysis and exploit the inner relationship between them. Further more, we study the relationship between Chinese stock indices and US stock indices in different time period. The results show that there is a higher level of mixed multifractal between SSEC and SZSE, and a lower level between China stock indices and NASDAQ. On the contrary, China stock indices has the highest level with DJIA which means that DJIA not only relates to US stock market, but also affects China stock markets.

1. Introduction

Multifractal has a far more complex scaling relation in the nature and requires a set of parameters to describe characteristics of complex systems. Multifractal is a development of the fractal, which was proposed by Mandelbrot in 1975 [1]. So far, the mathematical definition of fractal is still not strict, which only has a descriptive definition. Fractal is a rough or fragmented geometrical shape that can be subdivided into parts, each of which is a reduced-size copy of the whole. The multifractal has been extensively studied in recent years, for example, stock market [2], diffusion limited aggregation [3], turbulence [4], environment [5],central place system [6], filters [7], oil markets [8] and so on.

Recently mixed multifractal has generated great interest. Mixed multifractal analysis studies the simultaneous scaling behavior of finite measures. It provides the basis for a significantly better understanding of the local geometry of fractal measures. On the contrary, classical multifractal analysis studies the local scaling behaviour of single measure and it can only investigate the multifractality of one time series. Olsen [9] firstly established a general and unifying mixed multifractal theory of mixed Rényi dimensions and mixed coarse multifractal spectra. Mixed multifractal analysis thus combines local characteristics depending simultaneously on various different aspects of the underlying dynamical system with each other and provides the basis for a significantly better understanding of the underlying dynamics. Olsen's research builds a sound theoretical foundation for our study.

China stock market, as an emerging market, has been studied in recently years [10,12,13]. Previously, China stock market is a small and relatively closed market in the world. With China's rapid economic development, it is necessary for researchers to study the simultaneous scaling behavior between China stock market and global stock markets [14]. In this paper, we take US stock market as example.

In China and US stock markets, researchers found that both China and US stock index series are multifractal [10,11], but they focused on circumscribed single series. In this paper, we expand the classical multifractal analysis to mixed multifractal analysis to study the simultaneous scaling behaviour and relation of stock index series in China and America for the first time in the literatures. Mixed multifractal analysis gets three dimensional mixed spectra to study relationships of two stock markets. It provides the foundation for a significantly better understanding of the relationships between two stock indices because of concatenating local characteristics which are based simultaneously on various different aspects of the stock market.

The organization of this paper is as follows. In Section 2, we obtain the stock indices. In Section 3, we introduce the statistical moment scaling method to investigate the existence of multifractal in China and US stock markets. Then, in Section 4, we introduce the mixed multifractal analysis for the stock indices. In Section 5, we analyze the level of mixed multifractal in different stock index series by the obtained mixed multifractal spectra. Finally in Section 6, we make a conclusion.

* Corresponding author.
E-mail address: daimf@mail.ujs.edu.cn, daimf0225@163.com (M. Dai).

http://dx.doi.org/10.1016/j.chaos.2016.04.013

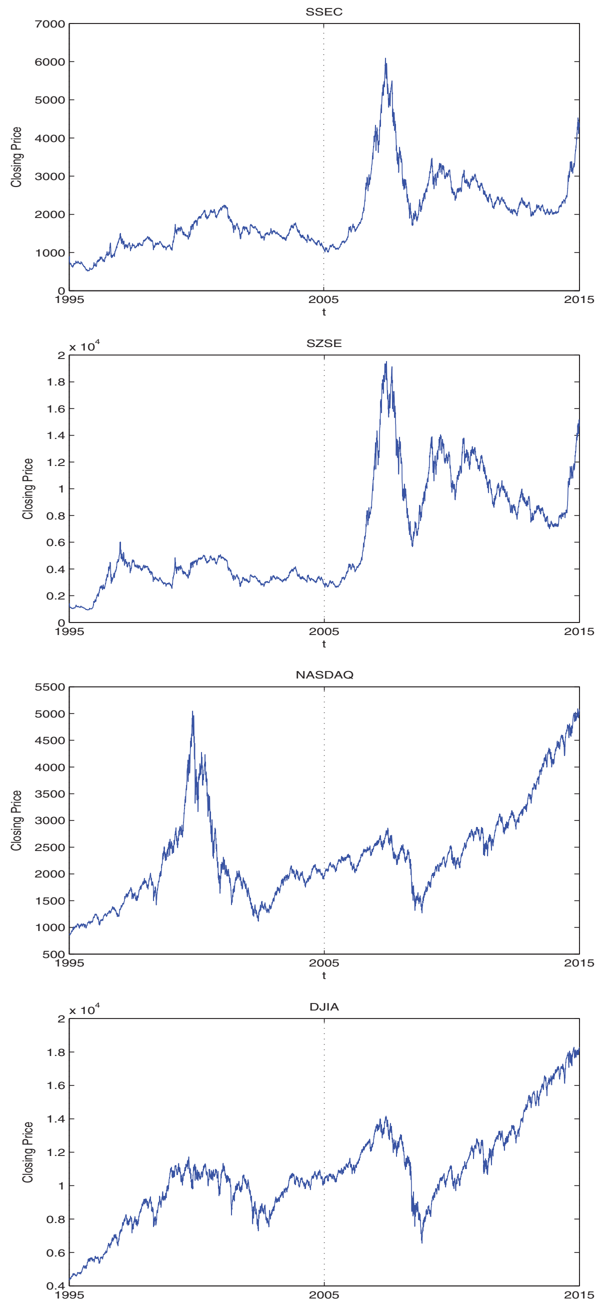

Fig. 1. Stock closing prices of SSEC, SZSE, NASDAQ, DJIA.

2. Data

The financial time series we use in this paper consists of four indices: two China stock indices, i.e., SSEC (Shanghai Stock Exchange Composite index), SZSE (Shenzhen Stock Exchange Composite index) and two US stock indices, i.e., DJIA, NASDAQ. We use stock closing prices of the four indices from May 18, 2005 to May 18, 2015 which are displayed in Fig. 1. We also use stock closing prices in another time period from May 18, 1995 to May 18, 2005 to make a comparison in Section 5. The original sample data are obtained from finance of yahoo (http://finance.yahoo.com/).

We concern on the daily stock price return and volatility series. For a giving stock market, denoting the stock market index as P_t, the volatility is defined as the absolute return, $x_t = |\log(P_t) - \log(P_{t-1})|$ [15]. The volatility series x_t is the time series we analyze in the following discuss.

3. Classical multifractal analysis of a single measure

In this section, we will use the statistical moment scaling method [17] to investigate the existence of multifractal in China and US stock markets.

Consider the volatility series x_t, $t = 1, 2, \ldots, n$ where n is the length of time series. Divide the series into $n_k = \text{int}(n/k)$ non-overlapping segments of equal length k. According to different values of k, we can calculate the average intensity in each interval i, which is a probability measure, denoted by

$$\mu_x(k, i) = \frac{N_x(k, i)}{\sum_{i=1}^{n_k} N_x(k, i)}, \quad (1)$$

where $N_x(k, i)$ is the sum of the x_t belonging to the ith interval.

It can be used to quantify the singular structure of $\mu_x(k, i)$ since Hausdorff multifractal spectrum of $\mu_x(k, i)$ describes the 'size' of the subset of points having the same scaling behavior. Hausdorff multifractal spectrum of measure $\mu_x(k, i)$ is defined by

$$f_H(\alpha) = \dim_H(E_\alpha),$$

where

$$E_\alpha = \left\{ i \in [1, n] : \lim_{k\to\infty} \frac{\log \mu_x(k, i)}{-\log k} = \alpha \right\},$$

α is the Lipschitz–Hölder exponent which reflects the local behavior of the measure. Raising $\mu_x(k, i)$ to power q, we can have the mixed covering moment scaling function of the measures μ_x by

$$M(k; q) = \sum_i [\mu_x(k, i)]^q. \quad (2)$$

Define the Rényi dimension

$$\tau(q) = \lim_{k\to\infty} \frac{\log M(k; q)}{-\log k}. \quad (3)$$

The Hölder exponent $\alpha(q)$ and multifractal spectrum $f_L(\alpha)$ are related to $\tau(q)$ via a Legendre transform [18],

$$\alpha(q) = d(\tau(q))/dq, \quad (4)$$

$$f_L(\alpha) = \inf_q (q\alpha(q) - \tau(q)). \quad (5)$$

Under very general assumptions, it have been proven that $f_H(\alpha) \le f_L(\alpha)$. For certain special classes of measures, we have the equality $f_H(\alpha) = f_L(\alpha) \equiv f(\alpha)$ [19].

We generate the multifractal spectra to analyze the multifractal behaviors in China stock market and US stock market. Fig. 2 displays the Legendre spectra $f(\alpha)$ for four stock series. The spectra all display the commonly observed bell shape, which means that the China stock indices and US stock indices both exhibit multifractal behaviors.

4. Mixed multifractal analysis

In this section, we will expand the statistical moment scaling method (introduced in Ref. [17]) to investigate the mixed multifractal of finite financial time series by mixed multifractal spectra. The mixed multifractal analysis studies the simultaneous scaling behavior of finite measures and provides the basis for a significantly better understanding of the mixed multifractal of two measures while classical multifractal analysis studies the local scaling behavior of one single measure.

Consider two time series x_t and y_t, $t = 1, 2, \ldots, n$ where n is the length of time series. Divide each series into $n_k = \text{int}(n/k)$ non-overlapping segments of equal length k. According to different values of k, we can calculate the average intensity in each interval i,