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Computing the topological entropy of continuous maps with at most three different kneading sequences with applications to Parrondo's paradox

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ABSTRACT

We introduce an algorithm to compute the topological entropy of piecewise monotone maps with at most three different kneading sequences, with prescribed accuracy. As an application, we compute the topological entropy of 3-periodic sequences of logistic maps, disproving a commutativity formula for topological entropy with three maps, and analyzing the dynamics Parrondo's paradox in this setting.

1. Introduction

Topological entropy is a non-negative number which is invariant by conjugacy for continuous maps defined on compact topological spaces. It was introduced in 1965 by Adler, Konheim and McAndrew, [2], and for continuous one dimensional maps it has been revealed as a way to "measure" the complexity of a discrete dynamical system, see e.g. [4, Chapter 4]. When the space is also metric, the topological entropy measures the exponential growth rate of the number of orbit segments distinguishable with arbitrarily fine but finite precision, [16]. Although the information that reveals about a system is an important value, in most cases, the computation of the topological entropy is not an easy problem. Milnor and Thurston, [20] developed the notion of kneading sequence for understanding iterated maps of the interval. Later, in 1989, Block, Keesling, Li and Peterson, [8] gave an algorithm to compute the topological entropy of unimodal maps with prescribed accuracy, involving the notion of kneading sequence. In 1992, [9], Block and Keesling extended the algorithm to compute the topological entropy of piecewise monotone maps with three pieces of monotonicity, again with prescribed accuracy. In [12] Cánovas and Muñoz gave a new step in this way introducing an algorithm to compute the topological entropy (with prescribed accuracy) of a particular class of piecewise continuous maps with more than 3 monotone pieces. Other algorithms based in different notions also have been introduced. Góra and Boyarsky [14] used a matrix-theoretic method for computing the topological entropy of continuous piecewise monotonic maps of the interval. Although more general, this algorithm was not as efficient nor as accurate as the algorithm in [8] for unimodal maps. Baldwin and Slaminka [7] presented an algorithm which is equivalent to the standard power method for finding eigenvalues of matrices. Steinberger [22] used a transition matrix associated with the transformation and gave a spectral theorem for this matrix. Recently, Amigó et al. [5] used the min–max symbols which are closely related to the kneading symbols to compute the topological entropy of twice differentiable multimodal maps. This algorithm was improved and simplified, (see [6]) proving that it is valid for continuous piecewise monotone maps.

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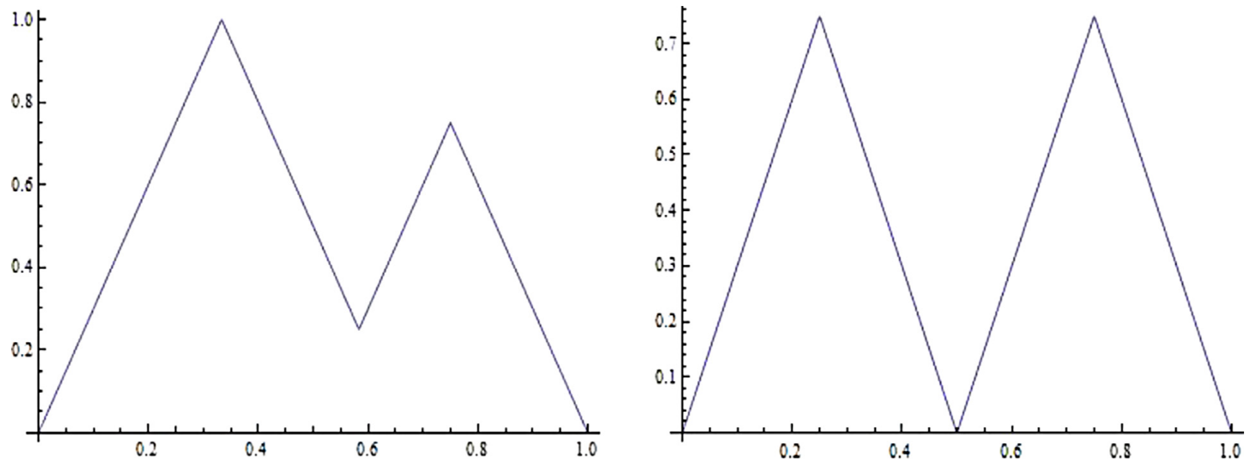

Fig. 1. We show the graph of several maps with constant slope 3.

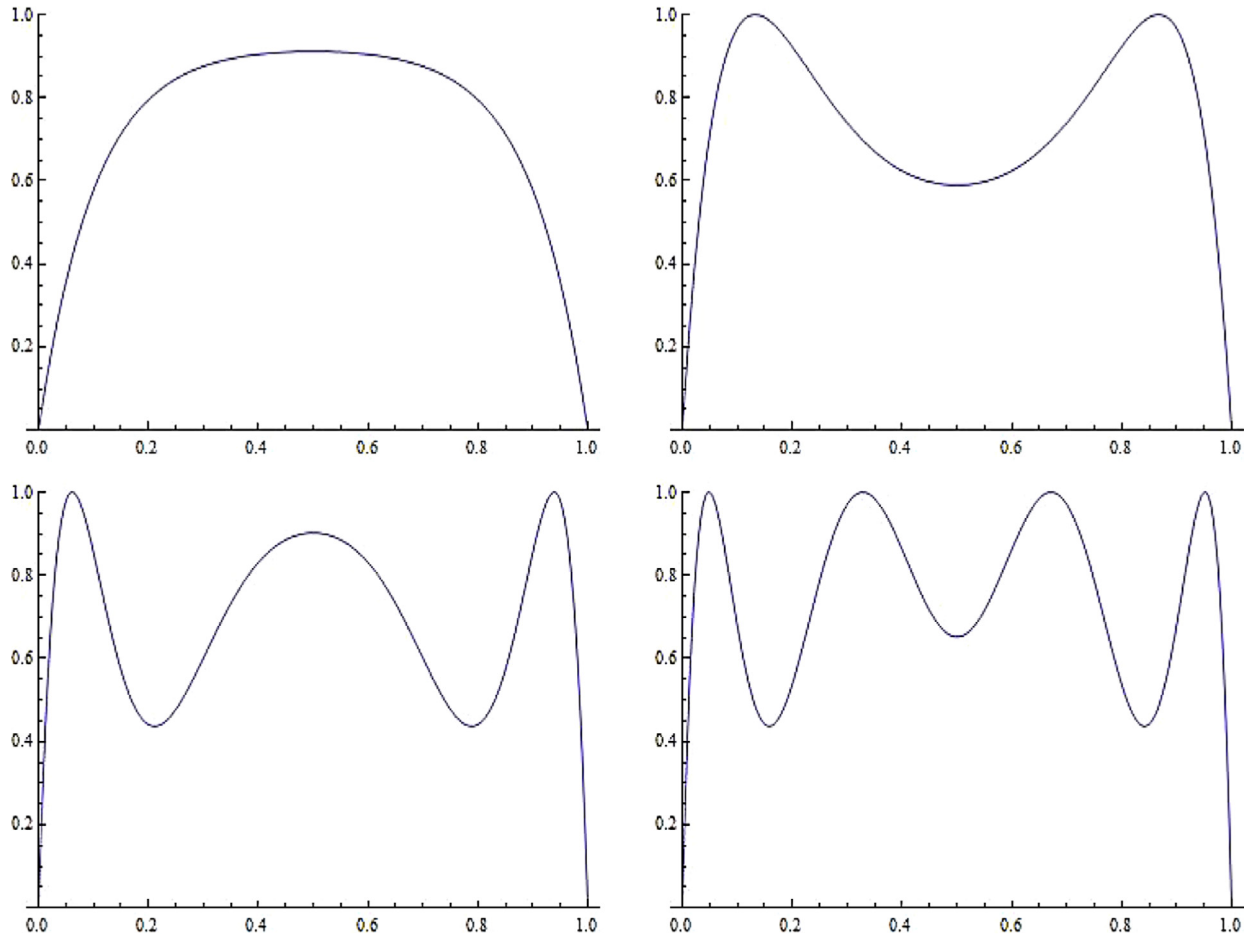

Fig. 2. We show the four cases described for the map $F_{a,b,c}$. This map is unimodal when $a = 4$ and $b = c = 1.5$, trimodal when $a = 4$, $b = 3.5$ and $c = 1.5$, it has 6 monotone pieces for $a = 4$, $b = 3.5$ and $c = 3$ and eight monotone pieces when $a = 4$, $b = 3.5$ and $c = 3.75$.