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Global modeling of aggregated and associated chaotic dynamics

Sylvain Mangiarotti [a,*], Flavie Le Jean [a], Mireille Huc [a], Christophe Letellier [b]

[a] Centre d'Études Spatiales de la Biosphère, CNRS-UPS-CNES-IRD, Observatoire Midi-Pyrénées, 18 avenue Édouard Belin, 31401 Toulouse, France

[b] Complexe de Recherche Interprofessionnel en Aérothermochimie, UMR6614 CNRS-Université de Rouen, site Universitaire du Madrillet, BP 12, 76801 Saint-Etienne du Rouvray cedex, France

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ABSTRACT

Spatially distributed systems are rather difficult to investigate due to two distinct problems which can be sometimes combined. First, the spatial extension is taken into account by monitoring the system evolution at different locations. Second, the dynamics cannot always be continuously tracked in time, and segments of data – sometimes recorded at different places – are only available. When the dynamics underlying a single marker is under consideration – as for instance the normalized difference vegetation index which can be used for assessing the vegetation canopy of a given area – a global model can be obtained from a single scalar time series built by aggregating the available time series recorded at different places and/or associating the segments of data recorded at different times (and possibly at different locations). We investigated how these two data preprocessing – common in environmental studies – may affect the model dynamics by using a system of spatially distributed Rössler systems which are phase synchronized or not.

1. Introduction

In science, one of the main goals is to obtain a set of differential, difference or discrete equations that reproduce the dynamics experimentally observed. In practice, a single variable of the studied area is commonly recorded. In such case one way to obtain a model of the dynamics is to use a global modeling technique which consists of projecting the experimental data, embedded in a reconstructed space spanned by derivatives or delay coordinates, onto a basis of functions [1–3]. This approach is especially interesting because it does not require any prior information on the algebraic structure of the governing equations [4,5]. Moreover, the amount of data required to obtain a global model is rather limited when the observability of the original state portrait is sufficiently good [5]. Global modeling was already successfully applied to observational or experimental data, as shown in various experimental situations [3,6–9].

Spatially distributed systems correspond to systems which can be viewed as resulting from a collection of local dynamics coupled with their neighborhoods over a spatially extended domain. In that case, the dynamics is rigorously governed by a set of partial differential equations. Such a system can be investigated by recording a few local variables (thus providing information on the local dynamics) or global variables which, most of the time, result from the integration of local quantities over a given domain. Attempts for extracting partial differential equations from local measurements is quite rare and requires a structure chosen *a priori* [10–12]. Extracting a global model from a global variable is not different from the common problem treated from measurements in a purely temporal system (for which the spatial extension can be neglected). Getting a global model from measurements in a spatially distributed system whose equations are not known *a priori* is therefore an open problem.

* Corresponding author. Tel.: +33 561556658; fax: +33 561558500.
E-mail address: sylvain.mangiarotti@ird.fr, sylvain.mangiarotti@cesbio.cnes.fr (S. Mangiarotti).

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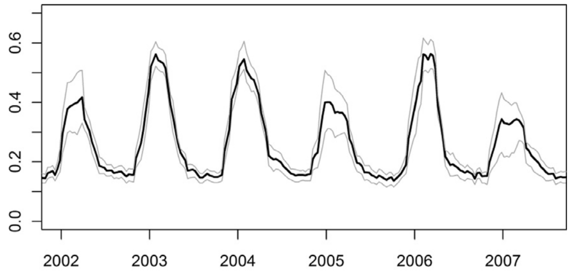
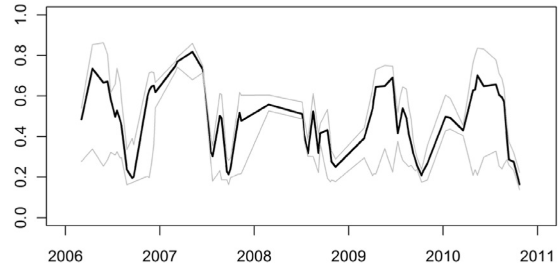
This problem is particularly important to tackle in the context of geosciences where it is often necessary to deal with data sets of different resolutions. Examples are numerous as, for instance, calibration of satellite observations that requires *in situ* measurements which often have very local representativeness compared to satellite data [13]. A similar problem is encountered when the recorded observational data does not necessarily have the same spatial resolution [14,15]. Data assimilation, the so-called process combining observations into models used for prediction, very often involves data from different spatial scales and different data types [16–19]. In order to do that, data from different locations can be aggregated, that is, combined in a weighted sum to provide a more meaningful variable: this is what is typically done – although implicitly – when a satellite with a low spatial resolution is used. When the time series are too short, the data can be associated (or concatenated). Although often used in geosciences, the influence of these two types of data preprocessings on global modeling was never investigated. Nevertheless, due to measurement constraints, scientists must work assuming that the underlying dynamics remains mostly unchanged by these data preprocessings (aggregation or association) [20,21].

Another particularity encountered in modeling land surface dynamics, agricultural and natural vegetation, soil humidity, etc. is that heterogeneity can be found at every scale, and for which *in situ* and remote sensors can provide information from very low (>1 km) to very high (<2.5 m) resolutions. When vegetation is observed from space at different places and resolutions, three different cases may be distinguished as follows. (i) The vegetation canopy is fully homogeneous in space and the dynamical states are simultaneously identical everywhere and at any resolution. The dynamics underlying aggregated time series is thus identical to the dynamics at any scale: the spatially distributed system can thus be viewed as a collection of (full- or phase-) synchronized elementary oscillators. This case is trivial and does not deserve any further investigation. (ii) The vegetation canopy is not spatially homogeneous as observed in regions where natural vegetation or crops are seasonally controlled by climatic conditions but it is still phase synchronized. This is the case in Sahelian regions controlled by the African monsoons [18,21] or in the semiarid regions of Morocco [9] (Fig. 1a). (iii) Heterogeneous and non synchronized vegetation can be observed where crops are locally controlled by irrigation, leading to independent behaviors no longer driven by climatic conditions. As an example, such a situation is observed in south of France where two time series are obviously not synchronized (Fig. 1b). These two different situations may be encountered at any spatial scale.

In practice, depending on the working conditions, several spatially distributed (possibly short) time series are aggregated into a single time series or a single scalar marker is measured: then, in both cases, a global model can be searched. If the possibility of getting a successful global model is clearly related to the underlying dynamics in both cases, it is also strongly related to the observability provided by the recorded time series [25]. The case in which multiple time series are aggregated or associated was never theoretically investigated and it is not known how these two data preprocessings can affect the quality of the global models if not, in certain cases, precluding the possibility to have a stable model. Such a fundamental question was triggered by the global models recently obtained from a time series aggregated from multi-temporal images observed from satellite [9].



(a) Safi province, Morocco



(b) Haute-Garonne province, France

Fig. 1. Time series of the normalized difference vegetation index observed from space [22]: (a) two time series estimated at distant locations in the Safi province of Morocco by averaging spatially the AVHRR data by using the GIMMS product (see [23] for details). (b) Two time series recorded at locations distant by 320 m at latitude 43°31′55″ and longitude 1°10′4″ in Haute-Garonne, south of France (the original 8 m resolution product of FORMOSAT-2 was orthorectified and free of cloud perturbation [24]).

Our first aim was therefore to investigate the effect induced by aggregating and associating different time series, independently of some observability problems. We will consider two types of spatially distributed systems: (i) a system Σ_c made of local dynamics which are driven by climate conditions (Fig. 1a), that is, which are phase synchronized, and (ii) a system Σ_i made of local dynamics controlled by irrigation (Fig. 1b), that is, non phase synchronized. From the measurement point of view, there are two different situations requiring data preprocessing before attempting a global model: (i) the spatial resolution is very high but the time series are short; the available data are thus *associated*. (ii) The spatial resolution is low or the data are very noisy; an aggregation is thus applied.

To avoid observability problem, we choose to first investigate the influence of these two data processings with the Rössler system which is known for its dynamical simplicity [26] as well as for providing variables offering a good or a poor observability of the original state space [25,27]. The spatial system will be thus made of nearly equivalent Rössler systems phase synchronized with a “master” Rössler system for simulating a system Σ_c or of independent nearly equivalent Rössler systems for reproducing a system Σ_i. To avoid a too large number cases, we will only “measure” the local dynamics by the same variable. A global modeling technique