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Binary generalized synchronization

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ABSTRACT

In this paper we report for the first time on the binary generalized synchronization, when for the certain values of the coupling strength two unidirectionally coupled dynamical systems generating the aperiodic binary sequences are in the generalized synchronization regime. The presence of the binary generalized synchronization has been revealed with the help of both the auxiliary system approach and the largest conditional Lyapunov exponent calculation. The mechanism resulting in the binary generalized synchronization has been explained. The finding discussed in this paper gives a strong potential for new applications under many relevant circumstances.

1. Introduction

Chaotic synchronization is known to be one of the fundamental phenomena, widely studied recently since it has both the theoretical and applied significance [1,2]. The study of the chaotic synchronization phenomenon is the central branch of the dynamical chaos theory dictated by its major fundamental significance accompanied with the broad spectrum of the practical applications including (but not limited to) the biological [3–5], physiological [6–9], chemical [10,11], controlling chaos [12,13] and information transmission [14–22] tasks.

There are the different types of chaotic synchronization revealed and studied recently such as phase synchronization [23,24], generalized synchronization [25,26], lag synchronization [27,28], complete synchronization [29,30], time scale synchronization [31,32], anti-synchronization[33], passive synchronization [19], adaptive synchronization [34,35], projective synchronization [36–38], generalized projective synchronization [39–41], modified generalized projective synchronization [21], etc. Among the different types of the synchronous behavior of chaotic systems the generalized synchronization (GS) [25,26,42] stands out due to its interesting features [43–46] and possible applications [18,47,48]. This kind of synchronous behavior means the state vectors of the interacting chaotic systems being in the generalized synchronization regime are related with each other. It has been observed in many systems both numerically [43,49–51] and experimentally [46,52,53].

The significant progress in the generalized synchronization studies has been achieved recently. In parallel with the revealing the mechanism being responsible for the generalized synchronization regime arising in the unidirectionally coupled chaotic oscillators [45,54,55], the concept of the generalized synchronization phenomenon has been extended to the mutually coupled systems and networks [56], as well as the relationship between the interacting systems has been clarified [57,58]. At the same time, generalized synchronization has been observed hitherto only for the *analog* systems (both the flows and maps), whereas phase synchronization and complete synchronization are known to be

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found recently in the *binary* systems whose signals contain only bits “0” and “1” (see Ref. [59] and Ref. [60], respectively).

Therefore, there is a fundamental problem, whether the type of synchronization with the properties of GS can exist for the systems generating the aperiodic binary signals. This problem is important from the point of view of both the generalized synchronization theory development and the generality of the chaotic synchronization phenomenon. In other words, the problem is the following: whether for the certain values of the coupling strength ε two coupled dynamical systems $\mathbf{x}(t)$ and $\mathbf{y}(t)$ generating the aperiodic signals $x(t)$ and $y(t)$ (where $x(t)$ and $y(t)$ take only the values “0” or “1”, t is time which may be both continuous and discrete) can be in the generalized synchronization regime.

In this work we report for the first time on the generalized synchronization between two unidirectionally coupled binary systems. Such type of synchronization we call as *binary generalized synchronization* (BGS).

2. Binary generalized synchronization

Let $\mathbf{x}$ and $\mathbf{y}$ are the drive and response systems (coupled unidirectionally with the coupling strength ε) generating the aperiodic binary signals $x = h(\mathbf{x})$ and $y = g(\mathbf{y})$, where h and g are some functions, with x and y taking only the values “0” or “1”. The dynamics of the systems $\mathbf{x}$ and $\mathbf{y}$ are governed by the evolution operators $\mathbf{H}[\cdot]$ and $\mathbf{G}[\cdot]$

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{H}[\mathbf{x}(t)] \\ \dot{\mathbf{y}}(t) &= \mathbf{G}[\mathbf{y}(t), \varepsilon x(t)] \end{aligned} \tag{1}$$

in the case of the flow systems and by

$$\begin{aligned} \mathbf{x}_{n+1} &= \mathbf{H}[\mathbf{x}_n] \\ \mathbf{y}_{n+1} &= \mathbf{G}[\mathbf{y}_n, \varepsilon x_n] \end{aligned} \tag{2}$$

in the case of the discrete maps.

The binary generalized synchronization takes place, when the condition

$$y = F[x] \tag{3}$$

is satisfied, where $F[\cdot]$ is the functional. The functional $F[\cdot]$ means the state of the response system signal y depends not only on the state of the drive signal x (whose value can be “0” or “1”) at the fixed moment of the continuous or discrete time, but on the pre-history of the evolution of the drive system (see works [57,58] where this point is considered in detail).

To detect the generalized synchronization regime in the unidirectionally coupled systems the different techniques have been proposed, e.g., the nearest neighbor method [25] or the conditional Lyapunov exponent calculation [26]. Among these techniques the auxiliary system approach proposed for the unidirectionally coupled chaotic oscillators may be generally considered as the most easy, clear and powerful tool to study the generalized synchronization regime in chaotic systems. Starting from the seminal paper of *Abarbanel* et al. [42], the auxiliary system approach has become de-facto the standard of generalized synchronization studies. Although the auxiliary system approach is not applicable for the mutual type of coupling [61] it is the very effective tool to detect the GS regime in unidirectionally coupled chaotic systems. The auxiliary system approach has been used in the plenty of theoretical and experimental works (see, e.g., [52,54,62,63]).

In our work we use two approaches mentioned above to detect the BGS regime, namely the auxiliary system approach and the largest conditional Lyapunov exponent calculation. The core idea of the auxiliary method approach consists in the parallel consideration of the dynamics of the response system $\mathbf{y}$ and the auxiliary system $\mathbf{z}$, whose dynamics is governed by

$$\dot{\mathbf{z}}(t) = \mathbf{G}[\mathbf{z}(t), \varepsilon x(t)] \quad \text{or} \quad \mathbf{z}_{n+1} = \mathbf{G}[\mathbf{z}_n, \varepsilon x_n]. \tag{4}$$

The auxiliary system must be completely identical to the response system $\mathbf{y}$, but it starts with the other initial conditions, i.e., $\mathbf{u}(t_0) \neq \mathbf{y}(t_0)$. If the generalized synchronization regime takes place, the signals of the response system, $y = g(\mathbf{y})$, and the auxiliary system, $z = g(\mathbf{z})$, become identical after the transient, since $y = F[x]$ and, simultaneously, $z = F[x]$. Obviously, in this case the condition $y = z$ should be fulfilled. On the contrary, in the case of the absence of generalized synchronization, the dynamics of the response $\mathbf{y}$ and auxiliary $\mathbf{z}$ systems are unrelated, $y \neq z$. Therefore, to detect the BGS regime, one has to compare the time series of the response and auxiliary systems with each other. To compare these time series, one can consider *the error*

$$E = y - z, \tag{5}$$

between values of the response and auxiliary system signals.

The presence of the generalized synchronization may be detected also with the help of the largest conditional Lyapunov exponent calculation. If the dimensions of the drive and response systems are equal to N, the *Lyapunov exponent spectrum* of the interacting systems is $\Lambda_1 \geq \Lambda_2 \geq \cdots \geq \Lambda_{2N}$. Since the drive system dynamics is independent on the behavior of the response system, this spectrum may be divided into two parts: exponents of the drive system $\Lambda_1^d \geq \cdots \geq \Lambda_N^d$ and conditional Lyapunov exponents [30,64] $\Lambda_1^r \geq \cdots \geq \Lambda_N^r$. The generalized synchronization regime takes place if and only if $\Lambda = \Lambda_1^r < 0$ (see [26] for detail).

3. Binary generalized synchronization in discrete systems

We have observed *the binary generalized synchronization* in two unidirectionally coupled systems whose equations read as

$$\begin{aligned} x_{n+1} &= H(\eta_{n+1}), \quad \eta_{n+1} = f(\eta_n, \lambda_d), \\ y_{n+1} &= H(\zeta_{n+1}), \quad \zeta_{n+1} = f(\zeta_n, \lambda_r) + \varepsilon \zeta_n^2 x_n, \end{aligned} \tag{6}$$

where x_n, y_n are the binary sequences under study, η_n and ζ_n are supposed to be the interior (hidden) variables whose dynamics is governed by the evolution operator

$$f(\xi, \lambda) = 1 - \lambda \xi^2, \tag{7}$$

λ_d and λ_r are the control parameters of the drive and response systems, respectively, ε is the coupling strength