

Controllability and synchronizability: Are they related?



Luis A. Aguirre^{a,*}, Christophe Letellier^b

^a Departamento de Engenharia Eletrônica, Universidade Federal de Minas Gerais – Av. Antônio Carlos 6627, Belo Horizonte 31270–901, MG, Brazil

^b CORIA UMR 6614 – Université de Rouen, Av. de l'Université, BP 12, F-76801 Saint-Etienne du Rouvray cedex, France

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ABSTRACT

In the two last decades the concept of observability has been formally linked to that of embedding in the context of nonlinear dynamics. Such a concept has been shown to play an important role in global modeling, data analysis and filtering, to mention a few examples. Preliminary results suggested that observability, at least in some cases, has some influence in synchronization problems. Could the dual concept of controllability also be important in such problems? In the context of synchronization, in general, the role played by controllability properties may not be as relevant as observability is for data analysis. In this work we compute controllability coefficients analogous to the observability ones, now established in the literature, and evaluate their importance in synchronization problems. Two benchmarks have been used in the simulations: the Rössler and the cord systems. The following schemes were investigated: synchronization to external sinusoidal force, complete replacement, uni- and bi-directional coupling of identical oscillators. The results discussed in this work show that controllability and synchronizability are not related in general.

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1. Introduction

Two fundamental properties of dynamical systems are observability and controllability. Such concepts were originally proposed by Kalman for linear systems [8] and subsequently extended to nonlinear systems, as pointed out in [6]. These concepts and related tools were conceived in the field known as control system theory.

Observability is closely related to embedding issues [CITEagulet/05 leteal/05preagulet/05 leteal/05pre, which are typically investigated in the field of nonlinear dynamics. The concept of observability has been recently used in a number of different applications in this field [10,14,16,20]. By contrast, the community of nonlinear dynamics has devoted less direct attention to the concept of

controllability, although some attempts have been made in the realm of complex networks [27].

Another important concept in the field of nonlinear dynamics is that of synchronization [19]. In a sense, bidirectional synchronization of two oscillators involves the concepts of observability and controllability. On the one hand, the state of one oscillator should be available to the other even if only one state variable is used in the coupling. This will guarantee that each oscillator “knows where the other one is in state space” in order to achieve synchronization. On the other hand, the coupling (control) terms should be effective in driving the oscillators towards the synchronization manifold [18]. Motivated by this understanding, some preliminary investigations were carried out using two Rössler systems bidirectionally coupled [11]. In fact, the correlation between observability properties, quantified by observability coefficients, and the capacity to synchronize by means of bidirectional dissipative coupling is quite amazing, at least in the case investigated in that paper.

* Corresponding author. Tel.: +55 3134094866.

E-mail addresses: aguirre@ufmg.br (L.A. Aguirre), letellie@coria.fr (C. Letellier).

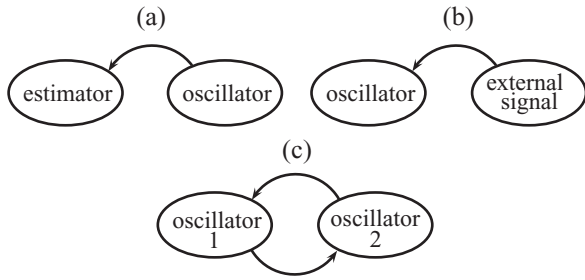


Fig. 1. Three general scenarios. (a) An oscillator being observed from a measured variable, (b) an oscillator being driven by an external signal, and (c) bidirectional coupling between two oscillators.

Unfortunately the narrow relationship between observability and synchronization that exists in the Rössler system does not apply to controllability in the general case. Also, the concept and usage of the term “controllability” is not standard in the literature. For instance, Whalen and colleagues [27] use the definition of controllability established in the field of nonlinear control [6], whereas other authors use alternative definitions [23] and some even equate the concepts of controllability and synchronization [26, p. 192].

Although observability does play a clear role in synchronization problems, it does not explain everything that is to it. Would controllability explain the remaining part? In fact, the requirements for a system to be totally controllable are essentially different to what is required for synchronization. A direct consequence of this is that the use of controllability coefficients might not be the right tool for investigating the capabilities of two systems to synchronize, i.e. synchronizability. The aim of this work is to investigate how pertinent are controllability coefficients as quantifiers of synchronizability.

The work is organized as follows. Background material is provided in Section 2. Section 3 details the benchmark systems used and present the controllability coefficients for each. The bench models are used in several synchronization schemes in Section 4 where indices of synchronization are used to try to assess if this property is related to the controllability coefficients. The main conclusions of the paper are discussed in Section 5.

2. Background

This section provides background material. Before going to technicalities, it will be helpful to first consider three general scenarios depicted in Fig. 1.

The oscillator in Fig. 1a is an autonomous system from which a variable is measured, which will be referred to as $s(t)$. The classic problem of state estimation is that of reconstructing the full state of such a system from $s(t)$ and the oscillator equations. This is possible only if the system is observable from the measured variable. For the sake of brevity we will just say that the system must be observable from $s(t)$. A closely related problem is that of finding a mathematical model for the oscillator from $s(t)$ [3]. It has been established that the ease with which a model is

found and the model performance are strongly influenced by certain observability properties [13].

By contrast, the oscillator in Fig. 1b is nonautonomous because it receives an external – and therefore time-dependent – signal. In an early paper in the field of synchronization [25], a periodic impulsive signal was added to the second equation of the Rössler system [21] and phase synchronization was verified. A particular case of the scheme shown in Fig. 1b is obtained when the external signal is actually one or more states of a second oscillator identical to the first one. In such a case it is common to add a negative feedback control term to one (or all) the equations of the first oscillator. Such a term is proportional, with gain K , to the difference between the state(s) of the second oscillator and the corresponding state(s) of the first one [9]. It has been argued [18] that if the system is synchronizable, the states of the oscillators used in the feedback control term will asymptotically approach each other and the coupled system will be equivalent to the complete replacement scheme originally proposed in [17]. Therefore, the scenario represented in Fig. 1b applies to several important cases.

In the scheme illustrated in Fig. 1a observability of the oscillator plays an important role whereas it plays none in Fig. 1b, because in the latter case nothing is measured from the system and observability is not even defined. On the other hand, since the oscillator in Fig. 1b is subject to an external (control) signal, one is left to wonder if controllability issues could turn out to be relevant in general for synchronization problems. Solís-Perales and co-workers have analyzed a specific situation using controllability and observability concepts: master–slave coupling, all equations of the slave oscillator are affected by the control input and only complete synchronization is considered [22]. Their analysis follows the “yes–no” dichotomy that permeates the control community and they establish conditions for a master–slave-coupled pair of oscillators to be feedback-linearizable on the synchronization manifold.

In the bidirectionally coupled scheme illustrated in Fig. 1c, one could even ask if observability and controllability could turn out to be important. However, it will be argued that if the aim is synchronization, controllability is not directly related in general. As a matter of fact, if the *whole system* is considered to be formed by coupled oscillators as in Fig. 1c, or as in Fig. 1b when the system includes the equations that produce the driving signal – and hence no longer is considered external), the system is autonomous, for which the concept of controllability is not even applicable. Of course, if a *single oscillator* in Fig. 1c is analyzed, controllability and observability of *that particular oscillator* are applicable concepts.

From the discussion in this section it is clear that there are several ways of *coupling* two oscillators, for instance, unidirectional or bidirectional coupling, in turn these can be applied to one or more variables, complete replacement, among others. Also, it is known that there are several *types* of synchronization, as for instance, phase synchronization, complete synchronization, generalized synchronization to mention a few [5]. Having decided which coupling to use and which type of synchronization to search for, there remains the issue of *quantifying* the synchronous

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