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Evaluating the transport in small-world and scale-free networks

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ABSTRACT

We present a study of some properties of transport in small-world and scale-free networks. Particularly, we compare two types of transport: subject to friction (electrical case) and in the absence of friction (maximum flow). We found that in clustered networks based on the Watts–Strogatz (WS) model, for both transport types the small-world configurations exhibit the best trade-off between local and global levels. For non-clustered WS networks the local transport is independent of the rewiring parameter, while the transport improves globally. Moreover, we analyzed both transport types in scale-free networks considering tendencies in the assortative or disassortative mixing of nodes. We construct the distribution of the conductance G and flow F to evaluate the effects of the assortative (disassortative) mixing, finding that for scale-free networks, as we introduce different levels of the degree–degree correlations, the power-law decay in the conductances is altered, while for the flow, the power-law tail remains unchanged. In addition, we analyze the effect on the conductance and the flow of the minimum degree and the shortest path between the source and destination nodes, finding notable differences between these two types of transport.

1. Introduction

In recent years, researchers from different disciplines have shown an increasing interest in the study of complex networks [1,2,4,3]. In particular, several approaches to classify and characterize complex networks have been proposed, which aim to help in the understanding of the networks operation and organization under different conditions [5,6], and very recently researchers have addressed the multiplex character of real-world systems [7]. As it is well known, a network is comprised of a set of nodes or vertices, and the set of links or edges that interconnect the nodes. The links can have a given direction and are known as directed; and in some cases they represent also a certain intensity in the connection which leads to networks with weighted links. The degree of a node is defined as the number of links that fall on it, when the links are not directed. However, in a directed network, the degree can be either interior (links coming into the node) or exterior (links coming out of the node). Moreover, the degree distribution $P(k)$, where k is the degree, allows describing the network connectivity, i.e., this distribution is obtained from plotting the frequencies against the degree, and characterizes the network; thus, the structure of $P(k)$ provides information on how the links are distributed [8,9]. For instance, it is known that random networks possess a characteristic connectivity scale [4], meaning that most of the

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nodes have an average number of links, that is described by a Poisson-like distribution. In many cases, real networks exhibit the small-world property, that is, the average path length is small and the clustering coefficient (local structure) is high [10]. A representative small-world model was proposed by Watts and Strogatz [10], which interpolates between a regular clustered network and a random graph; for intermediate configurations the small-world feature is observed. Moreover, there are networks where the distribution has no characteristic scale so they are called scale-free networks for which a significant number of nodes coexist with few links and few highly connected nodes known as hubs. For these networks, the degree distribution is given by a power law: $P(k) \sim k^{-\lambda}$ [9]. Recent studies on the transport properties in complex networks have reported that scale-free networks display better transport conditions than random networks, due to the presence of hubs [11,12]. Transport in non regular media, such as complex networks, provides an approach to the exploration of transport in many real conditions from electrical networks to the Internet [13–17]. Transport within a network consists of sending an entity from a specific node called the origin or source to another node called destination or sink. This problem can be stated as a flow problem to find the paths from source to destination for which it is possible to send as much flow as possible while satisfying capacity constraints on the links and flow conservation at the intermediate nodes [18,19]. Moreover, transport in many real situations involves the presence of friction. These cases can be modeled using analogies with electrical systems: a positive potential is assigned to the origin node while a zero potential is assigned to the destination node, and the links are considered as resistors. Based on the law of conservation of electrical charge, it is possible to estimate the current flow from the origin to the destination. On the other hand, a complex network can be classified according to the bias or the degree–degree correlations, i.e., if there is a bias in the connectivity between nodes with high or low degree, then the network has assortative or disassortative mixing [20]. Here, we are interested in evaluating the transport properties in small-world networks and the effect of assortative (disassortative) mixing on the transport in scale-free networks. Particularly, we focus on comparing the transport (with and without friction) in clustered and declustered small-world networks, by calculating the average conductance and the average flow on local and global scales; we find that for clustered networks and for both transport types, the best trade-off between local and global levels is observed for configurations with small-world topology, while for declustered networks, the transport improves globally and it is independent of the rewiring parameter. Besides, we also compare the effect of degree–degree correlations on the conductance and flow distributions in scale-free networks for three specific configurations. We observe a significant difference between the distributions for the three levels of assortative mixing. Our quantitative analysis also permits to test the effect on the conductance and the flow of the minimum degree and the shortest path between the source and destination nodes, finding notable differences between these two types of transport. The paper is organized as follows: In Section 2, a brief description of small-world and scale-free network models is given. Next, Section 3 describes the way the degree–degree correlations are introduced. The results and discussion are given in Section 4. Finally, in Section 5 some conclusions are presented.

2. Small-world and scale-free networks

Regular networks are the simplest model to describe the relationship between nodes since all nodes have the same degree; however, the model is not always appropriate to study real networks. An important model that interpolates between a regular and a random network is the Watts–Strogatz model (WS) [10]: starting with N nodes arranged in a ring with links to its k next–nearest neighbors (a k-regular network), *small-world* configurations can be created through a rewiring process that with some probability p reassigns links (when p is large enough, this rewiring process leads to a random network), creating shortcuts between distant sections of the ring. Thus, for $p = 0$ we have the case of a clustered WS network whereas for intermediate values of p, it is observed a high average clustering coefficient and short average path length, the main feature of the small-world network. A declustered WS model was proposed by Vragović et al. [21] and consists in starting with a regular ring with next–nearest neighbor connections and adding links from each site to only its nth neighbors [21]. In this way, the initial configuration has zero clustering coefficient. In our study, we consider that shortcuts are created with a random rewiring process as in the ordinary WS model. We notice that in the declustered model proposed in [21], the rewiring procedure considers only the more distant neighbors of a given site, while nearest-neighbor links are kept unchanged. In our case, we consider clustered and declustered WS networks with the same size and equal number of initial edges.

Moreover in many real systems the description of the connectivities is given by a power-law/scale-free distribution. An illustrative model to generate a scale-free network is that of Barabási–Albert where starting from a set of nodes with certain links, new nodes join according to the so-called preferential attachment process: nodes with more links are more likely to link new nodes [9]. The emerging network has a degree distribution of the power-law type $P(k) \sim k^{-\lambda}$ where the exponent, λ, depends on the type of network under consideration [1]. A more appropriate model to generate scale-free networks with a random mixing and a defined exponent, consists of using the Molloy–Reed algorithm on a set of N nodes [22]: in which, k_i copies for each node i are generated, where the probability of having a degree equal to k_i satisfies $P(k_i) \sim k_i^{-\lambda}$. These copies of nodes are randomly linked, without repeating links and avoiding self-loops.

3. Degree–degree correlations

Many complex networks exhibit tendencies in the assortative mixing between nodes. It has been observed in many cases that high-degree nodes tend to connect