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Influence of the nearest-neighbor connections on shaping weighted evolving network

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ABSTRACT

This paper proposes an extended local-world evolving network model consisting of global strength-driven preferential attachment for one central node, and local weight-driven preferential attachment for nearest neighbors of the central node. Analytical predictions and numerical simulations were executed for network evolutions and distributions. The obtained power-law behaviors display the same exponent functions as the ones in a classic model. More comparisons between these two models were made to investigate the structural differences that the nearest-neighbor connections result in. Compared with the counterpart, the proposed model shows a higher clustering coefficient, the varying average shortest path length and the significant hierarchical organization. our model is generally robustness and yet fragility, and is weaker in synchronizability than the counterpart. All those results are added to our understanding of how the rule of the nearest-neighbor connections affects the characteristics of weighted evolving network.

1. Introduction

The studies of social, biological, economic and numerous other networks have been added to our understandings of complex networks, which exhibit significant small-world property and scale-free behavior [1–6]. Many real-world networks are represented as weighted networks [7–11], thus, a amount of evolving weighted network models have been built to investigate the nontrivial correlation between edge weight and topological quantity [12–14]. One classic growing model for weighted networks was proposed by Barrat, Barthélemy and Vespignani (BBV) [15,16]. The BBV model is based on the mechanisms of strength preferential attachment and weight dynamical evolution. It displays scale-free behaviors for the distributions of node degree, strength and edge weight. Many extended models were later designed by adding new evolution rules including traffic-driven growth [17], spatial constraints [18], group-based preferential attachment [19] and accelerating growth [20–22].

In many real-world networks such as the world trade web [23] and the Internet on router level [24], it is difficult for one node to obtain global information about the entire network. A local-world evolving network model was therefore introduced to describe the preferential attachment mechanism on the local level [25]. To improve the low clustering coefficient, a triad formation step was later added into local preferential attachment [26,27]. The local world usually consists of randomly chosen nodes. However, taking a social network as the example, we find that one person more easily know s persons in the same community [28]. Suppose edge weight stands for the closeness between two persons, and a new individual *A* has established a relationship with the individual *B* who is famous (large value of strength). Then *A* has a high probability to know the individual *C* who is a close friend of *B* based on closeness between the individuals *C* and *B*. We thus build

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the model with one central node and its nearest neighbors as the local world, which is abbreviated as the NNLW model. The central node is selected by global strength-driven preferential attachment while neighbors of the central node to be connected are selected by local weight-driven preferential attachment.

The rest of the paper is organized as follows. Section 2 describes the evolutionary rules of the NNLW model. Section 3 provides the theoretical predictions and corresponding simulation results for the distributions and evolution of node degree, strength and edge weight. Section 4 makes full comparisons between the NNLW model and the BBV model by investigating the clustering coefficient, epidemic spreading and synchronization. Finally, Section 5 summarizes this work.

2. A local-world model with the nearest-neighbor preferential attachment

There are initially N_0 completely connected nodes and e_0 edges. A new node n with m edges is added at each time step, i.e., m existing nodes will be selected to connect the node n.

(i) Global strength-driven preferential attachment. One node i is firstly chosen from the existing network as the central node based on the probability of node strength:

$$\prod_{n\to i} = \frac{s_i}{\sum_{l\in all} s_l}. \tag{1}$$

(ii) Local weight-driven preferential attachment. $m-1$ nodes are selected from the nearest neighbors of the central node i (Ω_i) to connect to the new node n according to the probability of edge weight:

$$\prod_{n\to j} = \frac{w_{ij}}{\sum_{j'\in\Omega_i} w_{ij'}} = \frac{w_{ij}}{s_i}, \tag{2}$$

where j' is one of the nearest neighbors of the central node i.

The initial weight of each new edge (n, i) is w_0. Applying the same rule in the BBV model, our model rearranges the weights on all other edges departing from the node i based on weight ratio:

$$w_{ij'} \to w_{ij'} + \delta\frac{w_{ij'}}{s_i}, \quad j' \in \Omega_i. \tag{3}$$

The process leads to $s_i \to s_i + w_0 + \delta$, where δ is the weight increment of the new edge (n, i). We set $w_0 = 1$ and δ = constant in our experiment. After weight updating, the topological growth and weight dynamic run for another new node until the desired network size is reached.

3. Analytical calculations and numerical simulations

3.1. Degree and strength distributions

It is obvious that time equals the number of nodes added into the network, i.e., the network totally has $N = t + N_0$ nodes and $mt + e_0$ edges at time t. According to topological growth rules, when a new coming node n is added, one existing node i can be chosen either as the central node with probability given by Eq. (1), or as one of the nearest neighbors of the central node j by Eq. (2). The time evolution equation for k_i is:

$$\frac{dk_i}{dt} = \frac{s_i}{\sum_{l\in all} s_l} + (m-1)\sum_{j\in\Omega_i}\frac{s_j}{\sum_{l\in all} s_l}\frac{w_{ij}}{s_j} = m\frac{s_i}{\sum_{l\in all} s_l}. \tag{4}$$

According to the weight dynamic mechanism, the strength s_i of node i increases when the new node n connects either to i with the increment value of $1+\delta$, or to the nearest neighbors of node i with the increment value of $(w_{ij}/s_j)\delta$. Therefore,

$$\begin{aligned}\frac{ds_i}{dt} &= m\frac{s_i}{\sum_{l\in all} s_l}(1+\delta) + \sum_{j\in\Omega_i}\left(\frac{s_j}{\sum_{l\in all} s_l} + (m-1)\sum_{k\in\Omega_j}\frac{s_k}{\sum_{l\in all} s_l}\frac{w_{kj}}{s_k}\right)\left(\frac{w_{ij}}{s_j}\delta\right)\\ &= m\frac{s_i}{\sum_{l\in all} s_l}(1+\delta) + m\frac{s_i}{\sum_{l\in all} s_l}\delta = m\frac{s_i}{\sum_{l\in all} s_l}(1+2\delta).\end{aligned} \tag{5}$$

The total strength increased by each added edge is $2(1+\delta)$, implying $\sum_{i\in all} s_i(t) \approx 2m(1+\delta)t$. Eq. (5) can be solved with the initial condition $s_i(t=i) = m$, yielding $s_i(t) = m(t/i)^{(1+2\delta)/(2+2\delta)}$. According to Eqs. (4) and (5), $k_i(t) = s_i(t)/(1+2\delta)$ is obtained and displays a proportionality relation $s \sim k$.

Numerical simulations were performed to validate the obtained analytical predictions. Numerical results of time evolution for node strength in the top panel of Fig. 1 are consistent with the theoretical ones. The bottom panel of Fig. 1 shows that the relationship between the strength s_i and degree k_i always a linear, which is consistent with the predicted coefficient $1+2\delta$.

Suppose the time is uniformly distributed in $[0, t]$ when the node i is added into the network, and then strength probability distribution is written as:

$$P(s,t) = \frac{1}{t+N_0}\int_0^t \delta[s - s_i(t)]di, \tag{6}$$

where $\delta(x)$ is the Dirac delta function.

Because the infinite size limit $t\to\infty$, node strength distribution is $P(s) \sim s^{-\gamma_s}$ with $\gamma_s = \frac{3+4\delta}{1+2\delta} \in (2, 3]$. Node degree is linearly related to node strength, thus, the degree distribution $P(k)$ also has a power law form $P(k) \sim k^{-\gamma_k}$ with the exponent γ_k identical to γ_s. Fig. 2 shows the strength probability distributions $P(s)$ for different values of δ. The power-law behaviors of $P(s)$ are consistent with the theoretical predictions.

3.2. Weight distributions

In the process of network growth, the increase of edge weight w_{ij} is induced by a new connection between node n and node i (node j) with the following changing rate:

$$\frac{dw_{ij}}{dt} = m\frac{s_i}{\sum_{l\in all} s_l}\left(\delta\frac{w_{ij}}{s_i}\right) + m\frac{s_j}{\sum_{l\in all} s_l}\left(\delta\frac{w_{ji}}{s_j}\right) = \frac{\delta}{1+\delta}\frac{w_{ij}}{t}. \tag{7}$$

Suppose t_{ij} is the time when the edge (i,j) is established. The equation can be solved as $w_{ij}(t) = (t/t_{ij})^{\delta/(1+\delta)}$ with