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Observer-based adaptive control of chaos in nonlinear discrete-time systems using time-delayed state feedback

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ABSTRACT

A new approach to adaptive control of chaos in a class of nonlinear discrete-time-varying systems, using a delayed state feedback scheme, is presented. It is discussed that such systems can show chaotic behavior as their parameters change. A strategy is employed for on-line calculation of the Lyapunov exponents that will be used within an adaptive scheme that decides on the control effort to suppress the chaotic behavior once detected. The scheme is further augmented with a nonlinear observer for estimation of the states that are required by the controller but are hard to measure. Simulation results for chaotic control problem of Jin map are provided to show the effectiveness of the proposed scheme.

1. Introduction

The analysis and control of chaotic behavior in dynamic systems have been widely investigated in recent years [1–10]. To name a few, Ott et al., [1] developed a control scheme, called the OGY method, for stabilizing unstable periodic orbits embedded in chaotic attractor via a small control perturbation. Several extensions and successful applications of the OGY method have also been reported [4,11]. The OGY method and its extensions are, however, of state feedback control type, in that all states in control system are to be observed for control purposes [29]. Delayed state feedback control (DSFC) concept was developed, based on the difference between the current and the τ-time delayed output signals, to suppress the chaotic behavior in nonlinear continuous systems, where τ is a period of the stabilized periodic orbit [12–14]. The method, which is known for its simplicity [15,16], was further extended to nonlinear discrete-time systems [17,18]. Lyapunov exponents have also been used to quantify the chaos degree in complex dynamic systems [26,28], and effective computational tools for their calculation were proposed [19,27].

In this paper, a new method to adaptive control of chaos in nonlinear discrete-time systems is introduced. The focus is placed on systems that are originally bounded but can show chaotic behavior under parametric variation. The proposed approach employs an adaptive delayed state feedback control scheme. This scheme identifies chaos via on-line calculation of Lyapunov exponents. The calculation of Lyapunov exponents proposed earlier [20–22] will be used where an efficient QR-factorization technique [23] is employed for estimation of the Lyapunov exponents for systems whose time-varying parameters are to be identified via a generalized recursive least squares algorithm. The gain of the delayed state feedback controller is then adjusted, on-line, as a function of the maximum Lyapunov exponent. Variation of the control gain is consistent with the sign of the calculated Lyapunov exponent; it is assigned a very low value when the maximum Lyapunov exponent is negative and is set to a sufficiently high value when the maximum Lyapunov exponent becomes positive to suppress the chaotic behavior. The manner in which the gain is changed, is optimized for better response via an off-line genetic algorithm and the bounds on the controller gain are determined by invoking to the stability analysis of the entire closed-loop control system.

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Finally, the proposed controller is augmented with a nonlinear observer for those states of the system that are not measurable for feedback. An example is considered to substantiate the development made in this paper and comparison between the performance of the proposed controller and the one previously reported by the authors based on the gradient method [21] will be conducted. It is shown that the new controller takes less time to converge, and the convergence is less dependent upon the adaptation gain and does not lead to saturation of the control signal reported earlier in the application of gradient method.

2. Problem statement and controller development

Consider a discrete-time nonlinear dynamic system in the general form:

$$x(k+1) = f[x(k)] \tag{1}$$

where $k = 0,1,..$ is the number of the sampling instants, f is continuously differentiable, at least locally in a region of interest, and $x(k) \in R^n$. Here, we consider a class of nonlinear discrete-time systems which are originally not chaotic or unbounded but can show chaotic behavior when certain parameters of the system change during the system operation. Once the chaotic behavior occurs, at least one of the Lyapunov exponents of the system becomes positive. For stabilizing the unstable periodic orbits embedded in the chaotic attractor, a delayed state feedback control scheme (DSFC) is applied. Thus, the dynamic system (1), is augmented with control input sequence, $u(k) \in R^l (l \leqslant n)$ as follows:

$$x(k+1) = f[(x(k), u(k)] \tag{2}$$

where

$$u(k) = K[x(k-1) - x(k)] \tag{3}$$

and $K \in R^{l \times n}$ is the state feedback gain. A few remarks about the above control system are presented in the ensuing discussion.

Remark 1. Consider the linearized model of (2) as shown below:

$$\delta x(k+1) = A\delta x(k) + Bu(k) \tag{4}$$

Where, $\bar{x}$ is the fixed point of (1), $A = \frac{\partial f(x,u)}{\partial x}|_{x=\bar{x},u=0}, B = \frac{\partial f(x,u)}{\partial u}|_{x=\bar{x},u=0}$ and $\delta x(k) = x(k) - \bar{x}$.

Note that Eq. (3) is equivalent to the following equation:

$$u(k) = K[\delta x(k-1) - \delta x(k)] \tag{5}$$

Let $\delta z(k) = \delta x(k-1)$. Then, the linearized system of (2) around fixed point $\bar{x}$ can be written as follows:

$$\begin{bmatrix} \delta x(k+1) \\ \delta z(k+1) \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ I & 0 \end{bmatrix} \begin{bmatrix} \delta x(k) \\ \delta z(k) \end{bmatrix} \tag{6}$$

where I is a $(n \times n)$ identity matrix. As is seen, the local stability of (2) is reduced to the stability of the linear system (6). By studying Eq. (6), the dependence of the stability of the control system (2) on the value of K is clearly observed. As will be seen later, we will use the above approach to determine constraints on K. It is important to note that, if A has an odd number of real eigenvalues greater than one, then there does not exist K, such that (6) is asymptotically stable [17].

Remark 2. Characteristics of the nonlinear system (1) heavily depends on the manner in which its parameters change. In particular, the sign of Lyapunov exponents can change with parameter variations. When the maximum Lyapunov exponent becomes positive, it indicates a chaotic behavior. In that case, the controller (3) needs to act at full power to stabilize the system, otherwise it should stay inactive.

Thus, the elements of the state feedback gain, K, are chosen to change with the maximum Lyapunov exponent in the following manner:

$$K = [k_{ij}] = \left[\frac{\beta_{ij}}{1 + e^{-\alpha_{ij}\text{sign}(\lambda_{\max})}}\right] (i = 1, \ldots, l; \quad j = 1, \ldots, n) \tag{7}$$

In (7), parameters α_{ij} and β_{ij} are positive constants and $\lambda_{\max}$ is the maximum Lyapunov exponent of the control system. Proper tuning of parameters α_{ij} and β_{ij} is therefore essential to achieve a good performance. As will be described later in this paper, the tuning is done through an off-line optimization using a genetic algorithm.

3. Design of nonlinear state observer

The delayed state feedback control scheme requires that the states of the nonlinear system be accessible at all time. Thus, the controller must be augmented with an observer to make the controller realizable. Fig. 1 shows the block diagram of the proposed nonlinear observer. The nonlinear control system (2) with the measurable output vector $y(k) \in R^p$ is as follows: