Chaos, Solitons and Fractals 41 (2009) 2463–2470

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

journal homepage: www.elsevier.com/locate/chaos

Dynamical models of the human eye and strabismus

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ARTICLE INFO

Article history:
Accepted 15 September 2008

Communicated by Prof. G. Iovane

ABSTRACT

In this work, the applicability of a recently published dynamical model of the eye to the case of strabismus is investigated. Although the basic scheme of the original model remains valid, the simulation of the pathological dynamics requires a more suitable coverage of the space of the physiological rotations of the eye. This requisite is reached by developing the original model and by taking into account the contributions of connective tissues that were originally neglected. Possible wider fields of application of the model are then discussed.

1. Introduction

The problem of diagnostics is a very general one, that can be formally restated as the application of a filter that maps a healthy system into an "ill", "faulted", or "perturbed" one. This formalism can be equally be applied to a mechanical, industrial or natural "dynamical system" [6]. In order to be applied to real data, we have to assume that one or more time series are available with experimental data, and these can be considered observables of a dynamical system of which we do not necessarily know all the governing equations but which we can consider a good description of the system under study [7]. Therefore time series analysis becomes an important tool for diagnostic purposes and the time evolution of parameters that summarize the dynamics of a "human dynamical system", or one part of it, can therefore be used to monitor its health, or to distinguish between sane and ill subjects [1]. Of course many techniques can be applied to this purpose. Kurz et al. [8] investigated chaoticity of animal and human locomotive patterns and their biomechanical control parameters using Hurst and Lyapunov exponents. The largest Lyapunov exponent of cardiointervals time series was used by Pavlov et al. [9] as a discriminating parameter for cardio-vascular diseases. The recurrence quantification analysis of lung and tracheal sounds was used by Conte et al. [10] and Vena et al. [11] to characterize respiratory dynamics. In order to highlight the presence of skin lesions, Mastrolonardo et al. [12] proposed the combination of fractal analysis [13] with a tool coming from geostatistics, i.e. the variogram [14], aimed at characterizing the persistence – or the memory – of a time series and the dynamical system which is supposed to generate it. The eye movements were also characterized by extracting dynamical parameters from experimental time series. In particular, the human electroculographic activity (EOG) was used in the characterization of the transition from the awake to the asleep state [15].

A different approach was followed by Pascolo and Carniel [1], who tackled the problem of deriving, always starting from the experimental time series, a mechanical model of the human eye, aimed at the estimate of the level of muscular activation. In the model, real human eye movements studied in the laboratory are compared with the ones produced by a virtual eye described in kinematic terms and subject to the dynamics of six actuators, as many as the muscles devoted to the eye motion control. The definition of an error function between the experimental and the numerical response and the application of a suitable law that links activation and muscular forces are at the base of the methodology. The original

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0960-0779/$ - see front matter
doi:10.1016/j.chaos.2008.09.032

aim of [1] was the definition of a conceptual tool that could help the specialist in the diagnosis of potential physiological disturbances of saccadic and nystagmic movements. The work was in fact part of a collaboration between the Functional Mechanics Laboratory of the University and the Neurophysiopatology Laboratory of the ''S. Maria della Misericordia'' Hospital in Udine, Italy. Many other problems are related with the definition and characterization of the field of view, both in the perceptive and interactive terms, even when anomalies in the eye's movement control are present. In this work the dynamical three-dimensional model of the oculomotor apparatus proposed by [1] is analyzed and suitably updated in order to extend the field of applicability to the dominion of the great rotations and allowing the employment of the model even to the case of pathologies such as the strabismus. The new model, presented here in two variants, maintains the conceptual simplicity of its predecessor, that remains fully valid for small movements. The difference lies in the more accurate representation of the part of the extrinsic muscles which are wound around the eyeball. The modified models have been verified experimentally with respect both to the range of admitted ocular rotations and with respect to the entity of the muscular forces relative to extreme positions such as those arising in the diagnostic surveys in the evaluation of the strabismus.

2. Kinematic control of the eye and strabismus

The rotation of the eyeball is mainly controlled by six muscles, named extrinsic or extraocular muscles; they are the four recti muscles: superior, medial, inferior and lateral and the two oblique muscles: superior and inferior. All of these are able to produce both impulsive and durable actions.

The ocular system works in accordance with a logic of agonism–antagonism with the two eyes that constitute a single functional unit able to produce different type of motions: (1) *vestibulo-ocular movements*, aimed at stabilizing the images on the retina during the phasic movements of the head; (2) *optokinetic movements*, aimed to stabilize the images on the retina during the rotational movements of the head or to allow the eye to follow objects in motion when the head remains stationary; (3) *saccadic movements*, with the purpose of lining up rapidly the fovea and the peripheral visual target; (4) *pursuit movements,* which give the eyes the ability to follow a moving object around, although less accurately than the vestibulo-ocular reflex; (5) *vergence movements*, aimed at lining up the corresponding points in the retina of the two eyes with the target. The first four movements are said conjugates, in the sense that the two eyes make similar rotations in terms of amplitude and orientation; the fifth type of movement is said disjunctive, because the two eyes move in antiphase.

The activation of the six extrinsic muscles is entrusted to the oculomotor nerves: the one simply called *oculomotor nerve* (connected to the superior, inferior and medial recti and to the inferior oblique), the *trochlear* (connected to the superior oblique) and the *abducent* (connected to the lateral rectus). Possible injuries to the oculomotor nerves can induce paresis in the connected muscles, with effects that go from the deviation from the primary direction of aim, caused by the altered agonism–antagonism muscular relationship, to defects of rotation, up to the appearance of *diplopia* (double vision). The condition of altered alignment of the visual axes of the two eyes is said strabismus and it is measurable by the deviation from the visual axis in the non pathological case. The strabismus can affect both eyes (alternating strabismus, or bilateral strabismus, typical of children) or only one (monocular strabismus, or unilateral); it may be manifest (*tropia*) or latent (*phoria*), constant or intermittent.

Concomitant (non paralytic) strabismus is the one that is associated with angular deviation independent from the direction of aim, in contrast to the *incomitant (paralytic) strabismus*, for which the deviation is a function of the ocular orientation. Generally, the first is caused by the loss of coordination among the two eyes, the second from a paresis of the oculomotor muscles. The concomitant strabismus, more frequent in children and persistent in adult age if not adequately treated, may result in *esotropia*, or convergent strabismus, with the eyes rotating inwards, or in *exotropia*, or divergent strabismus, with the eyes rotating outwards. The incomitant strabismus is already observable in rest conditions and, increasing with an increasing rotation of the eyes, often leads to double vision, which may in turn produce dizzinesses and sense of nausea, up to producing difficulties in the deambulation.

A commonly adopted clinical protocol to measure strabismus is the Hess-Lancaster test, able to quantify the binocular alignment thanks to the employment of separated images for the two eyes. The result of the test is a diagram that shows the coordinates of image points that the patient was asked to target and of the points that the patient really targeted. The interpretation of the resulting offsets constitutes an important base in the planning of the surgery interventions for the strabismus, whose principals aims are in general, in order of importance, the restoration of the binocular vision and the attenuation of the diplopia, the recovery of the optimal collaboration in the cardinal positions and the reduction of possible postural anomalies.

The surgery generally works by modifying the ocular muscles through the methods of the recession, which involves moving the insertion of a muscle posteriorly towards its origin and has the purpose of reducing the muscular traction, and of the resection, aimed to increase it. A third surgery technique is the transposition, with the purpose of influencing the aiming direction, with the recession applied to couples of muscles.

3. Numerical models of the human eye

The starting point for the simulation of the human eye movements has been the three-dimensional parametric multi-body model proposed in [1]. This model is really constituted by the superposition of two submodels: the first, named