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Synchronization conditions for chaotic nonlinear continuous neural networks

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ABSTRACT

This paper deals with the synchronization problem of a class of chaotic nonlinear neural networks. A feedback control gain matrix is derived to achieve the state synchronization of two identical nonlinear neural networks by using the Lyapunov stability theory, and the obtained criterion condition can be verified if a certain Hamiltonian matrix with no eigenvalues on the imaginary axis. The new sufficient condition can avoid solving an algebraic Riccati equation. The results are illustrated through one numerical example.

1. Introduction

Recently, many efforts have mainly been devoted to the stability analysis and periodic oscillations of different kinds of neural networks [1–8]. It has been shown that such neural networks can exhibit complicated dynamics and even chaotic behavior if the parameters and time delays are appropriately chosen for the neural networks [9]. Up to now, there have been some studies in the synchronization of this class of chaotic neural networks with or without delays [10–15]. Cao et al. [10] analyzed synchronization of almost all kinds of coupled identical neural networks based on a simple adaptive feedback scheme. Zhou et al. [11] investigated lag synchronization of coupled chaotic delayed neural networks without noise perturbation by using adaptive feedback control techniques. Gu et al. [12] investigated complete synchronization of star-shaped complex networks using linear stability analysis. In [13], the authors discussed asymptotic synchronization of a class of neural networks with reaction-diffusion terms and time-varying delays. More recently, based on the Lyapunov functional method and Hermitian matrices theory, the authors [14] derived a synchronization criterion for coupled delayed neural networks, and Cheng et al. [15] applied the method into the synchronization for a class of neural networks with time-varying delays.

As a continuation of their previous published results, in this paper, a sufficient condition for the exponential synchronization of a class of chaotic nonlinear neural networks with time-varying delays is further exploited. The criteria are presented by employing the Lyapunov stability method and Hermitian matrices theory. A numerical example illustrates the applicability of the proposed approach.

1.1. Notations

In the sequel, we denote A^{T}, A^{-1} the transpose of, inverse of any square matrix A, respectively. We use $A > 0$; $(A < 0)$ to denote a positive- (negative-) definite matrix A; and I is used to denote the $n \times n$ identity matrix. The vector norm is taken to be Euclidian, denoted by $\|\cdot\|$. diag$(\cdot)$ denotes a block diagonal matrix. $\lambda(A)$ denotes the eigenvalue of a square matrix A. R^n and $R^{m\times n}$ denote, respectively, the n-dimensional Euclidean space, and the set of all $m \times n$ real matrices.

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2. Synchronization problem formulation

Based on the drive-response concept, the unidirectional coupled nonlinear neural networks are described by the following equations:

$$\dot{x}_i(t) = -\gamma_i(x_i(t)) + \sum_{j=1}^{n} D_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} D^{\tau}_{ij} f_j(x_j(t-\tau_j(t))) + J_i, \quad (1)$$

$$\dot{z}_i(t) = -\gamma_i(z_i(t)) + \sum_{j=1}^{n} D_{ij} f_j(z_j(t)) + \sum_{j=1}^{n} D^{\tau}_{ij} f_j(z_j(t-\tau_j(t))) + J_i + u_i(t), \quad (2)$$

for $i = 1, 2, \ldots, n$, where $n \geqslant 2$ denotes the number of neurons in the network, x_i is the state variable associated with the ith neuron. $D = (D_{ij})_{n\times n}$, $D^{\tau} = (D^{\tau}_{ij})_{n\times n}$ indicate the interconnection strength among neurons without and with time-varying delay $\tau_j(t) \geqslant 0$, respectively. The function f_i describes the manner in which the neurons respond to each other. While J_i is an external constant input; and $u_i(t)$ is unidirectional coupled term which is considered as the control input and will be appropriately designed to obtain a certain control objective. Furthermore, it is assumed that $\tau(t) = (\tau_1(t), \tau_2(t), \ldots, \tau_n(t))^T$, $\tau^* = \max(\tau_j(t))$ and $\sigma = \max(\dot{\tau}_j(t)) < 1$ for $j = 1, \ldots, n$ and $t \geqslant 0$, where τ^* and σ are constants, and the systems (1) and (2) possess initial conditions $x_i(t) = \phi_i(t) \in \mathscr{C}[-\tau^*, 0], R)$ and $z_i(t) = \varphi_i(t) \in \mathscr{C}[-\tau^*, 0], R)$, where $\mathscr{C}[-\tau^*, 0], R)$ denotes the set of all continuous functions from $[-\tau^*, 0]$ to R.

We further assume that the functions $\alpha_i(\cdot)$ and $f_j(\cdot)$ satisfies the following conditions.

(H1) Each function $\gamma_i : R \to R$ is locally Lipschitz and nondecreasing, and there exists a positive real a_i such that $\gamma'_i(x) \geqslant a_i$ for any $x \in R$ at which γ_i is differentiable. Let $A = \mathrm{diag}(a_i), i = 1, 2, \ldots, n$.

(H2) Each $f_j : R \to R$ is monotonic nondecreasing and globally Lipschitz, i.e. there exists a positive real $k_j > 0$ such that

$$0 \leqslant (f_j(x) - f_j(y))/(x - y) \leqslant k_j, \quad j = 1, 2, \ldots, N$$

for any $x, y \in R, x \neq y$.

Definition 1. The system (1) and the uncontrolled system (2) (i.e. $u \equiv 0$ in (2)) are said to be exponentially synchronized, if there exist constants $\beta(\alpha) \geqslant 1$ and $\alpha > 1$ such that

$$|x_i(t) - \tilde{x}_i(t)| \leqslant \beta(\alpha) \sup_{-\tau^* \leqslant s \leqslant 0} |\phi_i(s) - \varphi_i(s)| \mathrm{e}^{-\alpha t} \forall t \geqslant 0, \quad i = 1, 2, \ldots, n. \quad (3)$$

Constant α said to be the degree of exponential synchronization.

Exponential synchronization problem: The exponential synchronization problem considered here is to determine the control input u_i associated with the state-feedback for the purpose of exponentially synchronizing the two identical chaotic nonlinear neural networks (1) and (2) with the same system's parameters except the differences in initial conditions.

3. Some criteria for exponential synchronization

3.1. Controller design

Let us define the synchronization error signal $e_i(t) = x_i(t) - z_i(t)$, where $x_i(t)$ and $z_i(t)$ are the ith state variable of the drive and response neural networks, respectively. Therefore, the error dynamics between (1) and (2) can be expressed by

$$\dot{e}_i(t) = -\beta_i(e_i(t)) + \sum_{j=1}^{n} D_{ij} g_j(e_j(t)) + \sum_{j=1}^{n} D^{\tau}_{ij} g_j(e_j(t-\tau_j(t))) - u_i(t), \quad (4)$$

for $i = 1, 2, \ldots, n$, where

$$\beta_i(e_i(t)) = \alpha_i(x_i(t)) - \alpha_i(z_i(t)),$$
$$g_j(e_j(t)) = f_j(x_j(t)) - f_j(z_j(t)),$$
$$g_j(e_j(t-\tau_j(t))) = f_j(x_j(t-\tau_j(t))) - f_j(z_j(t-\tau_j(t))).$$

Here, from (H1) and (H2), we can obtain that the function $g_i(\cdot)$ satisfies $0 \leqslant e_i(t) g_i(e_i(t)) \leqslant k_i e_i^2(t)$, and according to the Lebourg Theorem [17], there exist $c_i \geqslant a_i$ such that $\beta_i(e_i(t)) = c_i e_i(t), i = 1, 2, \ldots, N$.

If the state variables of the drive system are used to drive the response system, then the control input vector with state feedback is designed as follows: