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Projective lag synchronization in chaotic systems

Guo-Hui Li *

Department of Communication Engineering, Shanghai University, Yanchang Road 149, Shanghai 200072, China

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ABSTRACT

In this paper, a new projective lag synchronization is proposed, where a driven chaotic system synchronizes the past state of the driver up to a scaling factor α. An active control method is employed to design a controller to achieve the global synchronization of two identical chaotic systems. Based on Lyapunov stability theorem, a sufficient condition is then given for the asymptotical stability of the null solution of an error dynamics. The effectiveness of the proposed schemes is verified via numerical simulations.

1. Introduction

Chaos synchronization has attracted considerably increasing attention and recently becomes an active area of research since it was suggested by Pecora and Carroll [1]. Originally, chaos synchronization refers to the state in which the drive and the response systems have exactly identical trajectories for $t \to \infty$, which is now called complete synchronization, identical synchronization, or conventional synchronization.

Over the last decade, following the complete synchronization, several new types of chaos synchronization of coupled oscillators have appeared, i.e., generalized synchronization [2–4], projective synchronization [5], phase synchronization [6], anti-phase synchronization [7], lag synchronization [8] and anticipating synchronization [9], etc. Generalized synchronization is defined as the presence of some functional relation between the states of response and drive, i.e., $y(t) = F(x(t))$. Projective synchronization is characterized by a scaling factor that two systems synchronize proportionally. Phase synchronization means that the phase differences among chaotic oscillators are locked within 2π, whereas their amplitudes remain chaotic and uncorrelated. Anti-phase synchronization is a phenomenon that the state vectors of the synchronized systems have the same amplitude but opposite signs as those of the driving system. In the lag synchronization, the state variable of slave is retarded with the time length of τ (τ is nonnegative real) in compared to that of master. In contrast to the lag synchronization, the anticipating synchronization is interpreted that the slave can anticipate the master's motion by synchronizing with its future state. Among these, great efforts have been devoted to the study of lag synchronization and anticipating synchronization, where the coupled systems follow identical phase space trajectories but are shifted-in-time relative to each other. Both lag and anticipating synchronization between the coupled oscillators can be obtained in a straightforward way using an explicit time-delay or memory.

In this paper, we will extend lag synchronization to study projective lag synchronization (PLS) in unidirectionally coupled time-delayed systems, where the response (slave) system's output lags behind the output of the driver (master) system proportionally. In Section 2, the concept of projective lag synchronization and scheme of synchronization are presented. Based on Lyapunov stability theory, Section 3 is devoted to synchronization for a class of chaotic systems using active control. Numerical simulations are also given in Section 3 to demonstrate the validity of theoretical results. Finally, brief conclusion remarks are drawn in Section 4.

* Tel.: +86 21 56332993.
E-mail address: shghlee@163.com.

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2. Theoretical results

2.1. Definition of projective lag synchronization

Inspired by the Ref. [10], we put forward a new kind of chaos synchronization, i.e., projective lag synchronization, which is described as follows.

Consider a drive system

$$\dot{x} = f(x), \tag{1}$$

and a response system

$$\dot{y} = g(y), \tag{2}$$

where $x = (x_1, x_2, \ldots, x_n)^T \in R^n$ and $y = (y_1, y_2, \ldots, y_n)^T \in R^n$ are state vectors, $f : R^n \to R^n$ and $g : R^n \to R^n$ are continuous nonlinear vector functions. Let $x(t, x_0)$ and $y(t, y_0)$ be the solutions of drive system and response system, where x_0 and y_0 are initial values. For simplicity, we denote $x(t, x_0)$ and $y(t, y_0)$ by $x(t)$ and $y(t)$, respectively.

Definition 1. If there exists a constant α ($\alpha \neq 0$) and a delay time $\tau > 0$, such that $\lim_{t\to\infty} \|x(t-\tau) - \alpha y(t)\| = 0$, then PLS between the system (1) and (2) occurs, and we call α 'scaling factor'.

Remark 1. PLS appears as a coincidence of shifted-in-time states of two systems, where the response system's output lags behind the output of the driver system proportionally with a scaling factor α. The synchronization manifold can turn out to be $x_\tau = \alpha y$, meaning that the response system follows the driver proportionally but is retarded in time relative to the driver, where $x_\tau \equiv x(t-\tau)$.

2.2. Scheme of projective lag synchronization

Suppose that the drive system (1) controls the response system (2). After a controller $u(t)$ is added, system (2) becomes

$$\dot{y} = g(y) + u(t), \tag{3}$$

where $u(t) = [u_1(t), u_2(t), \ldots, u_n(t)]^T \in R^n$ is the control vector.

Our goal is to design the controller $u(t)$ so that the projective lag synchronization manifold $x_\tau = \alpha y$ is globally attracting and asymptotically stable. Define error vector $e(t) = (e_1, e_2, \ldots, e_n)^T = \alpha y - x_\tau$, subtracting (1) from (3) yields the dynamical system of the synchronization error:

$$\dot{e} = \alpha\dot{y} - \dot{x}_\tau = \alpha g(y) + \alpha u(t) - f(x_\tau), \tag{4}$$

Construct a positive Lyapunov function:

$$V(e) = \frac{1}{2} e^T e. \tag{5}$$

Its time derivative along system (4) is

$$\dot{V}(e) = e^T \dot{e} = e^T(\alpha g(y) + \alpha u(t) - f(x_\tau)). \tag{6}$$

In Eq. (6), $u(t)$ is designed so that $\dot{V}(e) = e^T C_{n\times n} e$, where $C_{n\times n}$ is a diagonal negative definite matrix. Therefore, $\dot{V}$ is a negative definite function of e. By Lyapunov theorem of asymptotical stability

$$\lim_{t\to\infty} e = 0.$$

Hence, the error dynamical system (4) is globally asymptotically stable about the origin, implying that the two systems (1) and (3) are globally asymptotically projective lag synchronized.

3. Simulation illustrations

This section uses the Lyapunov stability theorem and an active control scheme to design a controller to synchronize the chaotic behavior of two identical Lorenz systems.

The drive system is given by

$$\begin{cases} \dot{x}_1 = \sigma(x_2 - x_1), \\ \dot{x}_2 = \gamma x_1 - x_1 x_3 - x_2, \\ \dot{x}_3 = x_1 x_2 - \xi x_3. \end{cases} \tag{7}$$

The unidirectionally coupled response system has the form