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# Simple robust technique using time delay estimation for the control and synchronization of Lorenz systems

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#### ABSTRACT

This work presents two simple and robust techniques based on time delay estimation for the respective control and synchronization of chaos systems. First, one of these techniques is applied to the *control* of a chaotic Lorenz system with both matched and mismatched uncertainties. The nonlinearities in the Lorenz system is cancelled by time delay estimation and desired error dynamics is inserted. Second, the other technique is applied to the *synchronization* of the Lü system and the Lorenz system with uncertainties. The synchronization input consists of three elements that have transparent and clear meanings.

Since time delay estimation enables a very effective and efficient cancellation of disturbances and nonlinearities, the techniques turn out to be simple and robust. Numerical simulation results show fast, accurate and robust performance of the proposed techniques, thereby demonstrating their effectiveness for the control and synchronization of Lorenz systems.

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#### 1. Introduction

A chaotic system is sensitive to initial conditions, highly nonlinear, irregular, and complex. Chaotic behaviors have been studied extensively since the first classical chaotic attractor was introduced by Lorenz [1]. Since the pioneering research of controlling chaos [2], some of the research moved from the pure analysis of chaos to the control and synchronization of chaos. In most engineering systems, chaotic behavior is undesirable, and the goal of chaos control is to suppress or remove chaotic behavior, and to provide the system with stable and predictable behaviors. On the other hand, in the applications of secure communications, biological systems, chemical reactions, and information processing, prescribed chaotic behaviors are wanted, and the goal of chaos synchronization is to make the chaotic states of the system to track the desired chaotic trajectory.

As an example of chaotic systems to be controlled, the Lorenz system is popular because the Lorenz system is simple among many chaotic systems, yet captures many features of chaotic dynamics [3,4]. Various methods have been introduced to control or synchronize the Lorenz system. For example, bang-bang control [3], sliding mode control [4], feedback linearization [5], adaptive control [6–8], backstepping control [9,10], neural networks [11,12], and others in [13]. Recently, fusions of aforementioned control methods have been carried out to achieve more sophisticated control performance. For example, fuzzy logic and adaptive control is merged in [14,15]; adaptive and backstepping technique are merged in [16]; advantages of the adaptive control, neural network and sliding mode control are combined in [17]; fuzzy adaptive sliding mode is used in [18]; and adaptive neural-fuzzy-network control is proposed in [19].

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To find out satisfactory solutions to chaos control and synchronization, in general, we should continue our research on the modeling and analysis of chaotic systems, and on fusing existing control methods, so that they become applicable to wider problem domain and provide more robust performance. In the mean time, when the parameters of chaotic systems are poorly understood, and if the practical implementation is considered, a viable alternative should be to pursue simplicity and transparency while preserving robustness. As for the simplicity, the control structure should have small number of terms in control input. As for the transparency, the control structure and the effect of tuning parameters should be transparent to designers.

As such a technique, we present a control technique using time delay estimation (TDE) method. The TDE was originally introduced to control robot manipulators [20–22]. The main idea of the TDE is to estimate unknown dynamics and disturbances by intentionally using time-delayed information. This estimation is used by our proposed technique to cancel the unknown dynamics and disturbances, while at the same time the desired dynamics is injected into the plant. The effectiveness of the TDE has consistently been crucial to the extraordinary robustness and simplicity demonstrated in [20–22] and other research works. Its effectiveness also motivates us to formulate the proposed technique for the control and synchronization of chaos dynamics, with expectations that the technique becomes simple in form, easy to implement, and yet robust.

There is another chaos control method intentionally using time-delay, which is called Pyragas method [23,24]. No prior goal (i.e. desired constant, periodic, or chaotic trajectory) can be specified in Pyragas method; the goal can only be achieved by trial-and-error in Pyragas method. On the contrary, the goal (desired constant, periodic, or chaotic trajectory) can be precisely specified and achieved through the proposed control using TDE technique.

This paper is organized as follows. In Section 2, we present TDE based control for regulations of the Lorenz system with matched and mismatched uncertainties. Section 3 presents simulation results for regulations of the Lorenz system. In Section 4, we design TDE based synchronizing technique for two different chaotic systems. Section 5 presents simulation results of the synchronization of two chaotic systems (the Lü system and the Lorenz system with parameter variation and disturbance). Finally, in Section 6, we give some concluding remarks.

#### 2. Regulation of the Lorenz system using TDE

The classical Lorenz system is described as

$$\dot{x}_1 = -\sigma x_1 + \sigma x_2, 
\dot{x}_2 = r x_1 - x_2 - x_1 x_3, 
\dot{x}_3 = x_1 x_2 - b x_3.$$
(1)

where  $x_1$  denotes the convective fluid motion,  $x_2$  denotes the horizontal temperature variation, and  $x_3$  denotes the vertical temperature variation;  $\sigma$ , b, and r are real positive parameters that represent the Prandtl number, a geometric factor, and the Rayleigh number, respectively.

The control of the Lorenz system is often realized by adding a control input u to the differential equation of state  $x_2$  [3]. A closed-loop experiment of the Lorenz equations with control input is given in [25]. Recently, bounded disturbance is considered in the differential equation of state  $x_1$ , and the differential equation of state  $x_2$  to emulate more practical situation [4].

Then, the controlled Lorenz system is expressed by

$$\dot{x}_1 = -\sigma x_1 + \sigma x_2 + d_1,\tag{2a}$$

$$\dot{x}_2 = rx_1 - x_2 - x_1x_3 + d_2 + u, (2b)$$

$$\dot{x}_3 = x_1 x_2 - b x_3,\tag{2c}$$

where  $d_1$  and  $d_2$  denote unknown disturbances, which are assumed to be continuous and bounded. As previous contributions [4,26], the control objective is to regulate  $x_1$  to a given constant  $x_{1r}$ . Setting  $x_1(t) = x_{1r}$  in (1), we can obtain the other equilibrium points of the states, which are  $x_2(t) = x_{1r}$ ,  $x_3(t) = x_{1r}^2/b$ . Thus, the goal is to design a control input u in order to regulate to a specific point  $P_r = (x_{1r}, x_{2r}, x_{3r}) = (x_{1r}, x_{1r}, x_{1r}^2/b)$ . The differential equation of state  $x_3$ , (2c), is internally stable when  $x_1(t)$  and  $x_2(t)$  converge to  $x_{1r}$ . So we will focus on the control of (2a) and (2b) from now on.

Defining the errors as  $e_1 = x_1 - x_{1r}$ ,  $e_2 = x_2 - x_{2r}$ , we can rewrite (2a) and (2b) as

$$\dot{e}_1 = -\sigma e_1 + \sigma e_2 + \eta_1,\tag{3}$$

$$\dot{e}_2 = f(e_1, e_2, e_3) + u,\tag{4}$$

where

$$\eta_1 = \sigma(x_{2r} - x_{1r}) + d_1,$$
 (5)

$$f(e_1, e_2, e_3) = re_1 - e_2 - x_{3r}e_1 - x_{1r}e_3 - e_1e_3 - x_{1r}x_{3r} - x_{2r} + rx_{1r} + d_2.$$

$$(6)$$

Note that since  $d_2$  was assumed to be continuous,  $f(e_1,e_2,e_3)$  becomes a continuous function, and because of this the approximation holds that  $f(e_1(t),e_2(t),e_3(t))\cong f(e_1(t-L),e_2(t-L),e_3(t-L))$  provided that L is sufficiently small. In other words,  $f(e_1(t),e_2(t),e_3(t))$  can be estimated by using  $f(e_1(t-L),e_2(t-L),e_3(t-L))$ . Let this estimation, the so-called TDE, be formally defined as

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