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Three-party quantum teleportation using thermal states in Heisenberg XX model with open boundary condition

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Abstract

Recently Yeo showed that thermal states in Heisenberg XX model with periodic boundary condition could be used for three-party quantum teleportation. However it is hard to implement the periodic boundary condition in spin chain. So instead of imposing the periodic boundary condition, we consider open boundary condition in Heisenberg XX model and investigate the possibility of using thermal states in Heisenberg XX model with open boundary condition. Using this way, we find the best fidelity conditions to three known protocols in three-party quantum teleportation. It turns out that the best fidelity in every protocol would be $\frac{2}{3}$.

0. Introduction

A quantum information can be teleported when classical communications (CC) and local operations (LO) are allowed to parties already sharing an entangled state. This is so-called 'quantum teleportation' and Bennett et al. [1] were the first to propose method between two parties.

Quantum teleportation scheme among three parties was also proposed by Karsson et al. [2]. In this scheme, Karsson et al. used GHZ state to teleport a quantum information among three parties. Bae et al. used three-qubit entangled states classified by Acin et al., for three-party quantum teleportation [3]. Jin et al. considered GHZ state and W one, when a receiver among three parties would be in an accelerated frame [4]. Recently Yeo showed the possibility of using thermal states in Heisenberg XX model with periodic boundary condition, for three-party quantum teleportation [5]. However in reality, it is hard to implement the periodic boundary condition in spin chain. So in this article, for three-party quantum teleportation, we consider Heisenberg XX model not with periodic boundary condition but with open boundary condition, which can be easily implemented in experiment. That is, we investigate the possibility of using thermal states of Heisenberg XX model with open boundary condition and find the conditions for best fidelity to these states in each protocol, for three-party quantum teleportation.

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This paper is organized as follows. In Section 1, we first review three-party quantum teleportation scheme. In Section 2, we consider three-party quantum teleportation using thermal state in Heisenberg XX model with open boundary condition and obtain the best fidelity conditions for these thermal states using three known protocols. In Section 3, we discuss the result and make a conclusion.

1. Quantum teleportation in three parties

Let us review the three-party quantum teleportation scheme proposed by Karsson et al. The quantum teleportation among three parties sharing a three-qubit entangled state is composed of three steps:

(1) A sender attaches an information qubit to the three-qubit entangled state and performs Bell basis measurement on his (or her) two qubits, (one is the information qubit and the other is his (or her) qubit entangled to other parties) and sends the measurement result j to the co-sender and the receiver. The Bell basis measurement makes use of the following projection operators: $|\Phi^+\rangle\langle\Phi^+|$ for $j=1$, $|\Phi^-\rangle\langle\Phi^-|$ for $j=2$, $|\Psi^+\rangle\langle\Psi^+|$ for $j=3$, and $|\Psi^-\rangle\langle\Psi^-|$ for $j=4$, where

$$\begin{aligned} |\Phi^\pm\rangle &= \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle) \\ |\Psi^\pm\rangle &= \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle) \end{aligned} \tag{1}$$

(2) The co-sender performs single qubit measurement at the stage, according to the sender's measurement result j, and sends the measurement result k to the receiver. The single qubit measurement uses the following projections: $|\mu^+\rangle\langle\mu^+|$ for $k=1$ and $|\mu^-\rangle\langle\mu^-|$ for $k=2$, where

$$\begin{aligned} |\mu^+\rangle &= \sin v|0\rangle + e^{i\kappa}\cos v|1\rangle \\ |\mu^-\rangle &= \cos v|0\rangle - e^{i\kappa}\sin v|1\rangle \end{aligned} \tag{2}$$

(3) Using the protocol provided in the secret when three parties are separated, the receiver performs local unitary operations according to the measurement results j and k. Then the party recovers the information state with a probability.

If $|\tau\rangle$ denotes the receiver's reconstructed state, then the success rate is measured by the fidelity of the original information state, $|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle$, and the $|\tau\rangle$, which is

$$\langle F\rangle = \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta \sum_{j,k} |\langle\psi|\tau\rangle|^2 \tag{3}$$

2. Three-party quantum teleportation using Heisenberg model

For three-party quantum teleportation we will use Heisenberg XX model which is described as

$$H = \sum \frac{J}{2}(\sigma_i^x\sigma_j^x + \sigma_i^y\sigma_j^y) + \frac{B}{2}\sigma_z \tag{4}$$

Here J is the coupling constant between spins and B is the strength of magnetic field. For three-party quantum teleportation Yeo considered the above model with periodic boundary condition. However in experiment it is not easy to implement Heisenberg XX model with periodic boundary condition. So we consider Heisenberg XX model with open boundary condition which can be easily implemented. The eigenvalues and the eigenvectors of the above model (4) with open boundary condition in three-qubit states are given in Table 1.

Using the eigenvectors and the eigenvalues, we can write the thermal density matrix of the system as follows:

$$\rho = \frac{e^{-\beta H}}{Z} \tag{5}$$

Here $\beta = \frac{1}{kT}$, $Z = \mathrm{tr}\, e^{-\beta H}$. The density matrix is shown to have some entangled behavior [6]. So we will use the density matrix to teleport an information qubit among three parties. Actually in three-party quantum teleportation there are