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Chaotic ranges of a unified chaotic system and its chaos for five periodic switch cases

Zheng-Ming Ge *, Kun-Wei Yang

Department of Mechanical Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu 300, Taiwan, ROC

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Abstract

In this paper, a unified chaotic system is studied in detail. Non-chaotic ranges within $\alpha \in [0,1]$ are found, where α is the constant parameter of the system. Chaotic range longer than $\alpha \in [0,1]$, $\alpha \in [-0.015, 1.152]$, is discovered, which is the extended chaotic range of unified chaotic system. Next, its chaos behaviors for five continuous periodic switch cases, $k\sin^2 \omega T$, $m \sin \omega t$, $0 \sim 1$ triangular wave, $-1 \sim 1$ triangular wave, and $0 \sim 1$ sawtooth wave, are presented.

1. Introduction

In recent years, chaos, chaos control and chaos synchronization became very interesting problems and have been widely studied [1–8]. Over the last decade, chaos synchronization has received considerable attention [9–17] due to its potential application in many areas such as secure communication, information process, biological systems, and chemical reactions.

New chaotic systems have been constantly proposed in the last 30 years. In 1999, Chen found a new chaotic attractor, the Chen system [18], which is dual to the Lorenz system and has a similarly simple structure but displays even more sophisticated dynamical behavior. Here, duality is in the sense defined by Vaněček and Čelikovský [19]: for the linear part of the system, $A = [a_{ij}]_{3\times 3}$, the Lorenz system satisfies the condition $a_{12}a_{21} > 0$ while the Chen system satisfies $a_{12}a_{21} < 0$. In 2002, Lü and Chen found another chaotic system, which satisfies the condition $a_{12}a_{21} = 0$ [20]. Very recently, Lü and Chen produced the third new chaotic system—unified chaotic system [21], which contains the Lorenz and Chen systems as two extremes and the Lü system as a special case. Recently, there are some results reported about the unified chaotic system [22–32].

In this paper, a unified chaotic system is studied in detail. Non-chaotic ranges within $\alpha \in [0,1]$ are found, where α is the constant parameter of the system. Chaotic range longer than $\alpha \in [0,1]$, $\alpha \in [-0.015, 1.152]$, is discovered, which is the extended chaotic range of unified chaotic system. Next, its chaos behaviors for five continuous periodic switch cases, $k\sin^2 \omega t$, $m \sin \omega t$, $0 \sim 1$ triangular wave, $-1 \sim 1$ triangular wave and $0 \sim 1$ sawtooth wave are presented. This paper is organized as follows. In Section 2, differential equations of motion and the description of the unified chaotic system are introduced. In Section 3, non-chaotic ranges within $\alpha \in [0,1]$ are found, where α is the original constant parameter of

* Corresponding author. Tel.: +886 35712121; fax: +886 35720634.
E-mail address: zmg@cc.nctu.edu.tw (Z.-M. Ge).

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the unified chaotic system. In Section 4, chaotic range longer than $\alpha \in [0,1]$, $\alpha \in [-0.015, 1.152]$, is discovered, which is the extended chaotic range of the unified chaotic system. In Section 5, the chaos of the unified chaotic system with $k\sin^2\omega t$ periodic switch are studied by Lyapunov exponents. The periodic time function $k\sin^2\omega t$ is used to replace the original constant parameter α of the unified chaotic system. In Sections 6–9, the chaos of the unified chaotic system with $m\sin\omega t$ switch, with $0 \sim 1$ triangular wave switch, with $-1 \sim 1$ triangular wave switch and with $0 \sim 1$ sawtooth wave switch are studied by Lyapunov exponents, respectively. In Section 10, conclusions are drawn.

2. The differential equations of motion and the description of the system

The unified chaotic system is described as follows:

$$\begin{aligned} \dot{x} &= (25\alpha + 10)(y - x), \\ \dot{y} &= (28 - 35\alpha)x + (29\alpha - 1)y - xz, \\ \dot{z} &= xy - \frac{8+\alpha}{3}z, \end{aligned} \tag{1}$$

where $\alpha \in [0,1]$. The system was said to be chaotic for any $\alpha \in [0,1]$ [24,32,33]. When $\alpha = 0$ the system is a classic Lorenz system, and when $\alpha = 1$ it becomes Chen system. Fig. 1 shows the Lyapunov exponents of the unified chaotic system. In three-dimensional space, the Lyapunov exponent spectra for a strange attractor, a two-torus, a limit cycle and a fixed point are described by $(+,0,-)$, $(0,0,-)$, $(0,-,-)$, $(-,-,-)$, respectively. In Figs. 2 and 3, we can see the phase portraits of the unified chaotic system with $\alpha = 0$ and $\alpha = 1$, respectively. Fig. 4 shows the phase portraits of case $(-,-,-)$ with $\alpha = -1$, and the phase portraits converge to a fixed point. Fig. 5 shows the phase portraits of case $(0,-,-)$ with $\alpha = 1.7$, and the phase portraits show a limit cycle.

3. The non-chaotic ranges within $\alpha \in [0,1]$ for the unified chaotic system

Unified chaotic system was told to be chaotic for any $\alpha \in [0,1]$ [33,34]. But we find that there exist some non-chaotic ranges within $\alpha \in [0,1]$. In order to confirm these ranges, three checking methods are used: Lyapunov exponents, phase portraits and power spectra. The Lyapunov exponent can be used to measure the sensitive dependence upon initial conditions of the states of the chaotic motion. It is the most reliable index for chaotic behavior. Different patterns of the solutions of the dynamical system, such as fixed point, periodic motion, quasi-periodic motion, and chaotic motion can be distinguished by them. Another valuable technique for the characterization of the solution of the system is the power

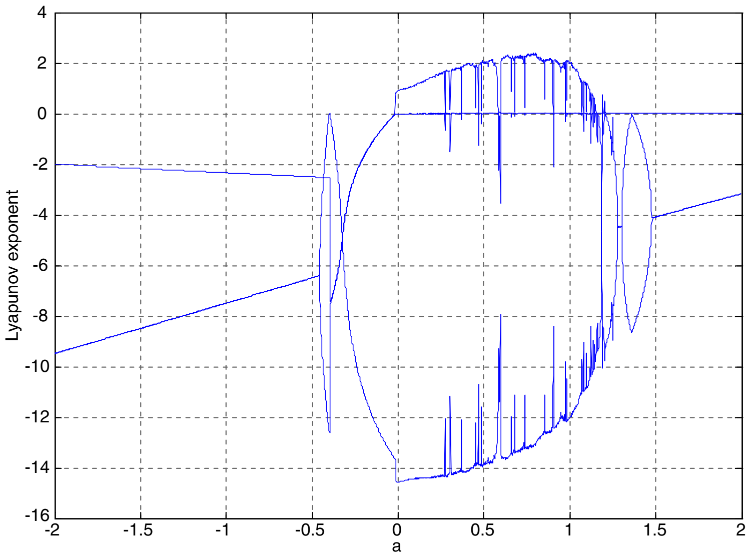

Fig. 1. The Lyapunov exponents of the unified chaotic system for α between 2 and -2.