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Chaos suppression based on adaptive observer for a $\mathscr{P}$-class of chaotic systems

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Abstract

A feedback approach is presented to suppress chaos in a $\mathscr{P}$-class of chaotic system. The approach is based on an adaptive observer; which provides estimated values of both the unmeasured states and the uncertain model parameters. A continuous-time feedback law is taken as suppressing force. The feedback law attains chaos suppression as the observer provides estimated values close to the actual state/parameter values along time. The proposed scheme is robust in the sense that suppression is achieved despite only some states are measured and uncertainties in parameters are compensated. Results are corroborated experimentally by implementation in chaotic circuits.

1. Introduction

Many researchers have been attracted to the control of chaotic systems due to its presence in different areas of knowledge. The chaos represents an interesting topic because irregular (erratic) oscillations can be undesirable in physical devices. Suppressing chaos can avoid damages to physical systems or attenuate oscillatory modes induced by chaos resonance. Moreover, chaos control can be exploited to deal with the current control problems like the chaos suppression on dc–dc converters, the use of multimode laser, the regulation of the fluid dynamics, the design systems for secure communications in internet, and some problems in biomedical sciences; for example, arrhythmias in heart. Several efforts have been made in order to solve problems for chaos suppression from the control theory; see, for example, [1–4]. Currently, the issues on chaos suppression are related to robust design and analysis of the feedback approaches. These issues are due to there are many systems where some partial information is available for feedback (measured states or parameters values) and the extremely high sensitivity of chaotic systems to the initial conditions and their parameter values.

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Here, the chaos suppression problem in a class of chaotic systems is addressed by designing adaptive observers. The class of systems includes jerk systems, which are characterized by the time-derivative (including abrupt varying) of the acceleration; i.e., $\dddot{x} = J(x, \dot{x}, \ddot{x})$. The motivation in designing chaos control for jerk equation is that its physical interpretation covers diverse engineering and natural systems. For example, jerk has been found in roll-drafting when stamped yarns are manufactured [7]; where the bundle of fibers can exhibits jerk due to discontinuity on the slope of the velocity profile near nip-line of the front roll. In nature, jerky phenomena can be found in universe expansion under Gauss–Bonnet gravity [8]; or on the description of the motion of charged particles in a forced field including the radiative reaction [9,10]. In fact, simple jerk systems are very useful for combining computations and analysis in space phase [11–13,19]. Some alternatives to overcome the problem of the chaos suppression are the design of feedback approaches based on adaptive structure and the observer-based schemes [4,5]. In regard adaptive schemes, some approaches have been proposed including robustness issues [6,14].

An strategy from adaptive-observer feedback is proposed in this contribution. The control scheme is robust in the sense that the unmeasured states and uncertain parameters are compensated and servo-control problem can be addressed. To design the proposed controller, an adaptive observer is firstly designed for the estimation of the state variable and actual parameters of the chaotic system. Then, using the estimated states and parameters, a dynamic output feedback control suppress the chaotic behavior. The theoretical results are corroborated experimentally by realization of a chaotic circuit representing a jerk system. The jerk system corresponds, in dynamical sense, to a sub-class dynamical systems ($\mathscr{P}$-class) and the results can be extended even for $\mathscr{P}$-class systems including continuous, non-differentiable functions (i.e., a sub-class of Lur'e systems). The manuscript is organized as follows. A brief description of the $\mathscr{P}$-class dynamical systems (jerk equation) is next section. The proposed algorithm is shown in third section. The implementation of the control algorithm is in Section 4; the experimental setup is included. Finally, some conclusions are given to close the text.

2. Chaotic systems of $\mathscr{P}$-class

Inspired in the Sprott's work [17–20], Malasoma [11] reported six dissipative chaotic jerk equations of the form $\dddot{x} = J(x, \dot{x}, \ddot{x})$; each of them defined by a polynomial function defined by three monomials included a single quadratic nonlinearity. These equations can exhibit chaotic behavior for various parameter values. Jerk systems are interesting because of they can be found in nature or manmade systems, and, additionally, can represent a sub-class of Lur'e systems when bursting is in velocity. In fact, jerk phenomena can be found in the drafting of textile fibers where the bundle of the fibers or in robots where some kinematic and dynamic constraints are implied. In addition to mechanical systems, other physical systems can also exhibit jerk phenomena; for instance the motion of charged particles in a forced field including a radiative reaction. Indeed, diverse nonlinear systems can be written as jerk-like equations. For example, after transformation, the Rossler model for homoclinic chaos in chemical reactions can be represented as a jerk equation. Actually, the jerk systems are very useful for combining analytical computations and dynamical analysis in phase space. This is particularly relevant since there is still no direct link between the algebraic structure of ODE's and the topology of the generated chaotic attractors. Thus, jerky systems are a good candidate to study control of chaotic systems. The unforced equations of the jerk systems have the form:

$$\dddot{x} + \alpha\ddot{x} + x - \phi(x, \dot{x}) = 0, \tag{1}$$

where $\phi(x, \dot{x})$ can be given, for example, by $\phi(x, \dot{x}) = \dot{x}^2$ or $\phi(x, \dot{x}) = x\dot{x}$. Eq. (1) can be written in a state representation as dynamical systems by defining $x_1 = x$, $x_2 = \dot{x}$, and $x_3 = \ddot{x}$. In this manner, Eq. (1) constitutes a family of systems with five monomials on their right-hand side; four coefficients that can be ± 1 by re-scaling of the three variables; and a single (scalar) independent control parameter at its right side. The unique parameter related to be the damping coefficient $\alpha > 0$. Thus, for instance, we have a collection of dynamical systems modeled by (1) [11], on which, among others, the following systems are included:

$$\Sigma_1 : \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = -\alpha x_3 - x_1 + x_2^2, \end{cases} \tag{2}$$

$$\Sigma_2 : \begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = -\alpha x_3 - x_1 + x_1 x_2, \end{cases} \tag{3}$$