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Single-mode control and chaos of cantilever beam under primary and principal parametric excitations

A.F. El-Bassiouny *

Mathematics Department, Faculty of Science, Benha University, Benha, Kalubia 13518, Egypt

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Abstract

A non-linear control law is proposed to suppress the vibrations of the first mode of a cantilever beam when subjected to primary and principal parametric excitations. The dynamics of the beam are modeled with a second-order non-linear ordinary-differential equation. The model accounts for viscous damping air drag, and inertia and geometric non-linearities. A control law based on quantic velocity feedback is proposed. The method of multiple scales method is used to derive two-first ordinary differential equations that govern the evolution of the amplitude and phase of the response. These equations are used to determine the steady state responses and their stability. Amplitude and phase modulation equations as well as external force–response and frequency–response curves are obtained. Numerical simulations confirm this scenario and detect chaos and unbounded motions in the instability regions of the periodic solutions.

1. Introduction

Many physical systems are described by differential equations that can be separated into one part containing linear terms with constant coefficients and a second part or non-autonomous terms. Accordingly, the system is said to be weakly non-linear or weakly non-autonomous, or both, are referred to as perturbations. The perturbation terms are generally identified by means of a small parameter ε. A weakly non-linear system is called quasi-linear. If the system is quasi-linear, then a solution is commonly sought in the form of a power series in the small parameter ε. This is the so-called analytical approach, and the techniques used to obtain time-dependent solutions are known as perturbation methods. Most systems are non-linear in nature or tend to become non-linear under certain conditions. The analysis of non-linear systems shows quantitative as well as qualitative differences, among other things, limit cycles, jump phenomena, saturation, subharmonic, superharmonic, supersubharmonic and combination resonances. A major cause of this disagreement stems from ignoring the effect of non-linear terms; see for example Refs. [1–5].

Hatwall et al. [6] and Hatwall [7] attached a pendulum to a single-degree-of-freedom system consisting of a mass and a restoring spring to act as a passive vibration absorber. Zavodney et al. [8] studied the response of a model that includes quadratic and cubic geometric non-linearities. They found that stable limit cycles can exhibit quasi-periodic and chaotic motions. Zavodney and Nayfeh [9] investigated the dynamics of a cantilever.

* Address: Quassim, Onaizah, P.O. Box 3137, Saudi Arabia.
E-mail address: tef_elbassiouny@yahoo.com

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Beam carrying a lumped mass. They modeled the structure with cubic geometric and inertia non-linearities. They conducted experiments and reported results that were in general agreement with the theory. Nayfeh et al. [10] investigated chaos and instability in a power system. Palkovics and Venhovens [11] analyzed stability, Hopf bifurcations, and chaotic motion in controlled wheel suspension systems. Virag and coworkers [12] investigated stability of auto-parametric resonance in an externally excited system. Asfar and Masoud [13] performed the control of parametric resonance using a Lanchester-type damper and obtain successful vibration suppression and bifurcations control. El-Dib [14] analyzed stability for a parametric non-linear Schrodinger equation containing the first-order spatial and the first-order temporal derivatives of a complex conjugate type. Stepan and Haller [15] considered quasi-periodic oscillations in robot dynamics. Nayfeh and Nayfeh [16] investigated a single-degree-of-freedom system with cubic non-linearities to an amplitude modulated excitation whose carrier frequency is much higher than the natural frequency of the system. Rega [17] studied non-linearity, bifurcation and chaos in the finite dynamics of different cable models. Anderson et al. [18] improved the model proposed by Zavodney and Nayfeh [9] and considered the effect of quadratic damping on the response of the system. Their theoretical results agree very well with the experiment. Moiola et al. [19] dealt with the more general forced non-linear system under delay control in the case of non-linear structural vibrations with a time delay in damping. Deryugin et al. [20] develop an analytic approach to the problem of the behavior stabilization of dynamical systems by parametric perturbations. Nayfeh and Asfar [21] analyzed the response of a bar constrained by a non-linear spring to a harmonic excitation. Yanng et al. [22] studied combination resonances in the response of the Duffing oscillator to three-frequency excitation. El-Dib [23] used the method of multiple scales to determine a third-order solution for a cubic non-linear Mathieu equation. The perturbation solutions are imposed on the so-called solvability conditions in the non-resonance case yield the standard Landau equation. Several types of a parametric Landau equation are derived in the neighborhood of five different resonance cases. El-Bassiouny and Abdelhafez [24] investigated predication of bifurcations for external and parametric excited one-degree-of-freedom system with quadratic, cubic, and quartic non-linearities. Ashour and Nayfeh [25] analyzed a non-linear adaptive vibration absorber to control the vibrations of flexible structures. The absorber is based on the saturation phenomenon associated with dynamical systems possessing quadratic non-linearities and two-to-one internal resonance. Huang and Wang [26] studied the steady-state analysis for a class of sliding mode controlled systems using describing function method. Ng and Rand [27] investigated the effect of non-linearities on a parametrically excited ordinary differential equation whose linearization exhibits the phenomenon of coexistence. Eissa and El-Bassiouny [28] investigated analytical and numerical solutions of a non-linear ship motion. Alsaif [29] investigated the dynamic response of a non-linear semi-definite mechanical structure. It consists of a cantilever beam subjected to a harmonic excitation at the base; two hinged rigid plates are attached at the other end. El-Bassiouny and Eissa [30] studied the dynamics of a single-degree-of-freedom structure with quadratic, cubic and quartic non-linearities to a harmonic resonance. Mahmoud and Farghaly [31] investigated chaos control of chaotic limit cycles of real and complex Van der pol oscillators. Wu and Li [32] used two versions of multiple scales and Kryov–Bogoliubov–Mitropolsky (KBM) methods to obtain the asymptotic solutions of a class of strongly non-linear oscillators with slow varying parameters. A comparison of the two methods is made to show that the multiple scales method is equivalent to the KBM method for the first-order approximation. Maccari [33] used the asymptotic perturbation method to the study of the parametrically excited and weakly damped Boussinesq equation in an infinite wall. Elhefnawy and El-Bassiouny [34] analyzed the non-linear stability and chaos in electrohydrodynamics. Cao [35] studied primary resonant optimal control for homoclinic bifurcations in single-degree-of-freedom non-linear oscillators. Jing and Wang [36] analyzed complex dynamics in Duffing system with two external forcings. El-Bassiouny [37] used the method of multiple scales to investigated principal parametric resonances of non-linear mechanical system with two-frequency and self-excitations. Nayfeh et al. [38] and Sridhar et al. [39] studied the response of uniform hinged-clamped beams to primary resonances of either the first or the second mode. Chen et al [40] studied the response of a uniform beam to a primary resonance of either the first or the second mode. Lau et al. [41] studied the response of a uniform hinged-clamped beam to a primary resonance of the first mode and a combination subharmonic resonance of the first two modes. Tuer et al. [42] proposed active control strategies based on internal resonances for controlling the free vibrations of oscillatory systems. They introduced a controller taking the form of a second-order system that is coupled to the plant via quadratic or cubic non-linear terms. Upon proper tuning of the controller's natural frequency, the non-linear terms act as an energy bridge and a state of exchange of energy is established between the plant and controller, resulting in a beat in the response of the total system. When the controller absorbs most of the plant's energy, a damping mechanism is activated to prevent the energy from returning back to the plant. Oueini and Golnaraghi [43] implement this internal resonance control strategy developing a circuit that emulates the equations of a controller with quadratic coupling terms. Chin and Nayfeh [44] investigated the non-linear responses of suspended cables to primary resonances. Chang analyzed [45] subharmonic resonances in harmonically excited rectangular plates with one-to-one internal resonance. El-Dib [46] investigated a theoretical analysis of the parametric harmonic response of two