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From lag synchronization to pattern formation in one-dimensional open flow models

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Abstract

In this paper, the relation between synchronization and pattern formation in one-dimensional discrete and continuous open flow models is investigated in detail. Firstly a sufficient condition for globally asymptotical stability of lag/anticipating synchronization among lattices of these models is proved by analytic method. Then, by analyzing and simulating lag/anticipating synchronization in discrete case, three kinds of pattern of wave (it is called wave pattern) travelling in the lattices are discovered. Finally, a proper definition for these kinds of pattern is proposed.

1. Introduction

Emergence is one of the most important properties in complex systems. Many efforts were devoted to it in past twenty years. Holland [1], a famous leading exponent in this field, studied the emergence in complex systems mainly by computer science, and obtained some significant results. Here, we discuss the problem based on the theory of nonlinear dynamics. We conjecture that the representation form of emergence in complex systems is pattern emergence from the viewpoint of dynamics, and the mechanism causing emergence of pattern formation is various synchronization among the basic elements composing the complex system.

Since Kaneko et al. put forward three basic models—the CML, GCM, and open flow models to describe dynamics in lattices in 1989, lattices dynamics has attracted a great deal of attention and has become a kind of classical model to research temporal–spatial dynamical behavior. Kaneko [2] studied in detail various patterns in $\mu\epsilon$-space of the following system as $f(x) = \mu x(1-x)$:

$$\begin{cases} x_0(n) = x_1(n), \qquad x_{N+1}(n) = x_N(n), \\ x_i(n+1) = (1-\epsilon)f(x_i(n)) + \frac{\epsilon}{2}[f(x_{i-1}(n)) + f(x_{i+1}(n))], \quad i = 1, \ldots, N. \end{cases} \tag{1}$$

and discovered that there exist plenty of dynamical phenomena. And Liu et al. [3] further discussed the complete synchronization of the system (1). After that, complete synchronization in two-dimensional and three-dimensional lattices

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system with locally coupled limit-cycle [4,5] and oscillators [6] was studied analytically. In fact, the complete synchronization in CML model represents trivial pattern of the systems in physics space. Moreover, cluster synchronization in 2D CML model was investigated by Belykh et al. [7,8]. All these investigations for synchronization, however, seem to take no consideration of delayed coupling.

In recent years, delayed coupling has been paid much attention, because time delay often exists in signals travelling between elements (lattices), such as we observed in biological and economic systems. In Ref. [9], Masoller and Zanette studied the synchronization of the systems of the form

$$\begin{cases} x_{n+1} = \alpha f(x_n) + \beta f(x_{n-n_1}) + g(x_n), \\ y_{n+1} = \alpha_s f(x_n) + \beta_s f(y_{n-n_1}) + g(y_n) + \eta f(x_{n-n_2}), \end{cases} \tag{2}$$

where α, β, α_s, β_s are parameters, and η is coupling strength. They show that, by suitably tuning parameters (α_s, β_s), two distinct kinds of synchronization are possible—one is lag synchronization, the other is anticipating synchronization. Li et al. [10] investigated the complete synchronization in small-world networks of phase oscillators with delayed coupling. And in Ref. [11], Li Chunguang, Chen Guanrong studied the complete synchronization in complex dynamical networks of the form

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} G_{ij}Ax_j(t-\tau), \quad i = 1, 2, \ldots, N, \tag{3}$$

where $f: R^n \mapsto R^n$ is a continuously differentiable function, $x_i = (x_{i1}, \ldots, x_{in})^\top \in R^n$ is a state variable of node i for $i = 1, 2, \ldots, n$, $G = (G_{ij})_{N\times N}$ is the coupling matrix of the network, and $A = (a_{ij})_{N\times N}$ is an inner-coupling matrix of the nodes. However, we find that these works do not further analyze what pattern is produced due to lag or anticipating synchronization.

On the other hand, it is known that various patterns can be emerged from complex systems. For example, the results are observed by experimentation that electric activity in neural networks objectively shows the behavior of pattern emergence and that there are various patters, such as target wave, spiral wave and scroll wave, on excited media. There were a lot of discussions about pattern formation, most of which were focused on numerical research. The discussions about mechanism of pattern formation are very few.

In fact, the pattern should be geometric representation of correlation among basic elements composing the complex system in physics space. The various correlation among basic elements composing the complex system can be described by various synchronized phenomena. So we believe, by constructing proper models, this idea can be confirmed through theoretic analysis and numerical simulation. We conjecture the results obtained by this idea should give profound understanding on pattern emergence in complex systems.

In this paper, we consider synchronization in 1-D open flow models with time-delayed coupling among lattices. By analytic study, for two cases—dynamics on lattices being discrete or continuous, we respectively obtain the sufficient condition for existence of globally stable lag/anticipating synchronization in the system. Further, through theoretic analysis and numerical simulation of lag/anticipating synchronization in discrete case, we discover that there are three kinds of wave pattern travelling in the lattices. And we propose a definition for these kinds of wave pattern, which possess significance in many fields, such as biology, science of brain, etc.

The layout of the paper is as follows. In Section 2 and Section 3, we respectively investigate lag/anticipating synchronization among lattices in discrete and continuous cases, and prove globally asymptotic stability of synchronization manifolds under some given conditions. In Section 4, based on the theory in Section 2 and numerical simulation, we take the Logistic map as an example to analyze various kind of lag/anticipating synchronization in parameter space. Then we try to define three kinds of wave pattern in the lattices. In the final section, we discuss the feasibility of further research on lag/anticipating synchronization.

2. Global lag synchronization in 1-D discrete open flow model with unidirectional delayed coupling

We consider a 1-D open flow model with unidirectional delayed coupling of the form

$$\begin{cases} x_0(n+1) = f(x_0(n)), \\ x_i(n+1) = f(x_i(n)) + \epsilon[f(x_{i-1}(n-1)) - f(x_i(n))], \quad i = 1, 2, \ldots, N, \end{cases} \tag{4}$$

where $x_i = (x_{i1}, x_{i2} \ldots, x_{im})^\top \in R^m$ is a state variable of lattice i, $i = 0, 1, 2, \ldots, N$, and n is discrete time. $f: \Omega \mapsto \Omega$ is a continuously differentiable function with $\Omega \subset R^m$ being compact and $0 \leqslant \epsilon \leqslant 1$ is coupling strength. Obviously, free