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Designing synchronization schemes for chaotic fractional-order unified systems

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Abstract

Synchronization in chaotic fractional-order differential systems is studied both theoretically and numerically. Two schemes are designed to achieve chaos synchronization of so-called unified chaotic systems and the corresponding numerical algorithms are established. Some sufficient conditions on synchronization are also derived based on the Laplace transformation theory. Computer simulations are used for demonstration.
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1. Introduction

It has been found that many systems in interdisciplinary fields can be described by fractional differential equations, for example, viscoelastic systems, dielectric polarization, electrode–electrolyte polarization, and electromagnetic waves [1,2]. Studying fractional-order differential systems is becoming an active research field.

There are material differences in many aspects, e.g. qualitative properties, between ordinary differential equation (ODE) systems and fractional-order differential systems. That most of properties or conclusions of the ODE systems cannot be simply extend to the case of the fractional-order differential systems is a known fact [3,4]. Now, a problem we ask is when an ODE system is chaotic, under what condition the corresponding fractional-order system is also chaotic. More exactly, for what orders, the fractional-order system is chaotic. Furthermore, given a chaotic fractional-order differential system, how to design a scheme such that synchronization of many such systems is achieved.

The latter problem even in the ODE case has not yet been thoroughly solved since feasibility of a designed synchronization scheme depends heavily on the considered single system. It is no doubt that studying a fractional-order system is much more difficult than doing an ODE system whatever in either theoretical analysis or numerical simulation. Therefore, designing synchronization schemes to fractional-order differential systems is a more challenging task, which is the main concern of this paper.

It is known that some fractional-order differential systems behave chaotically, e.g., the fractional Duffing system [5], the fractional Chua system [6], the fractional Chen system [7], the fractional Lü system [8]. These fractional-order systems can be also synchronized [5–8]. In this paper, we further investigate the synchronization problem of chaotic fractional-order differential systems both analytically and numerically. By taking the unified chaotic system [10] as an

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example, we design two schemes to realize synchronization of two identical unified systems and establish numerical algorithms. In fact, our design scheme and numerical algorithms can been easily extended to other similar fractional-order differential systems.

2. Definitions and systems

There are several definitions of a fractional-order differential system. In the following, we introduce the most common one of them:

$$D_*^{\alpha}x(t) = J^{m-\alpha}x^{(m)}(t) \quad \text{with} \ \alpha > 0, \tag{1}$$

where $m = \lceil \alpha \rceil$, i.e., m is the first integer which is not less than α, $x^{(m)}$ is the m-order derivative in the usual sense, and J^{β} ($\beta > 0$) is the β-order Reimann–Liouville integral operator with expression

$$J^{\beta}y(t) = \frac{1}{\Gamma(\beta)}\int_0^t (t-\tau)^{\beta-1}y(\tau)\,d\tau. \tag{2}$$

Here Γ stands for Gamma function, and the operator D_*^{α} is generally called "α-order Caputo differential operator" [9].

In 2002, Lü et al. [10] introduced a new chaotic system by unifying the Lorenz, Chen and Lü systems . The system is described by

$$\begin{cases} \dot{x} = (25\alpha + 10)(y - x), \\ \dot{y} = (28 - 35\alpha)x - xz + (29\alpha - 1)y, \\ \dot{z} = xy - \frac{\alpha+8}{3}z, \end{cases} \tag{3}$$

where $\alpha \in [0, 1]$. It has been shown that system (3) is chaotic for all $\alpha \in [0, 1]$. Obviously, when $\alpha = 0.0$, $\alpha = 0.8$ and $\alpha = 1.0$, the system are the Lorenz system, the Lü system and the Chen system, respectively. An interesting numerical result is that when $\alpha \in [0.0, 0.8)$ and $\alpha \in (0.8, 1.0]$, the corresponding chaotic attractors are similar to the Lorenz attractor and the Chen attractor, respectively.

Now, let us introduce its fractional version as follows:

$$\begin{cases} \dfrac{d^{q_1}x}{dt^{q_1}} = (25\alpha + 10)(y - x), \\ \dfrac{d^{q_2}y}{dt^{q_2}} = (28 - 35\alpha)x - xz + (29\alpha - 1)y, \\ \dfrac{d^{q_3}z}{dt^{q_3}} = xy - \dfrac{\alpha + 8}{3}z, \end{cases} \tag{4}$$

where $d^{q_i}/dt^{q_i} = D_*^{q_i}$ $(i = 1, 2, 3)$, its order denoted by $q = (q_1, q_2, q_3)$ is subject to $0 < q_1, q_2, q_3 \leqslant 1$, and $\alpha \in [0, 1]$. We have found that for a set of parameter values: $\alpha \in [0, 1]$ and $q = (0.985, 0.99, 0.99)$, the fractional-order unified system can display chaotic attractors as shown in Figs. 1–3, respectively.

This paper will propose two different schemes to realize chaos synchronization of the fractional-order unified systems. And computer simulations justify the proposed schemes for the same fractional orders.

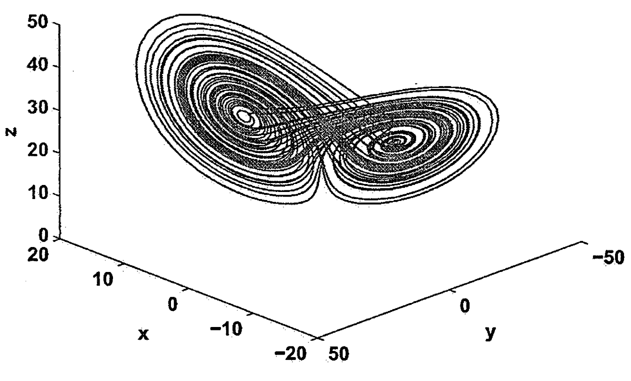

Fig. 1. The Lorenz-like chaotic attractors in the fractional-order unified system: $\alpha = 0.2$, where $q = (0.985, 0.99, 0.99)$.

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