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Non-linear waves in complex plasma

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Abstract

The combined effects of the non-adiabatic dust charge fluctuation and the inhomogeneous of equilibrium dust density and charging on non-linear dust acoustic waves are presented theoretically. By the perturbation method, a variable coefficients Korteweg–de Vries–Burgers equation governing the non-linear waves is derived. It is shown that both solitary waves and shock waves (both oscillatory and monotone shock) can exist in this complex plasma.

1. Introduction

Solitons and shock waves would be the fundamental non-linear coherent structures in dusty plasmas and that has been paid a great deal of interest in recent years [1–11]. The presence of the charged dust grains in plasma give rise to complexity and enriched the non-linear excitations in the system. It is also important to note that inhomogeneity and dissipativity are general features of this complex (dusty) plasmas, which exists in both laboratory and space. In an inhomogeneous dusty plasma, the equilibrium quantities, such as dust charging and dust density, are non-uniform and vary spatially. Experimental data demonstrate that most plasma equilibrium parameters exhibit strong variations that are especially pronounced along the discharge tube. Hence, consideration of dissipative properties of the non-linear waves in inhomogeneous complex plasma should be an important and profound issue [12–17]. Comparatively little research has been done about such problem. In the previous investigation about the waves in inhomogeneous dusty plasmas, the dust grains has been treated either as having uniform equilibrium dust density, or with constant charging, or just considering the adiabatic dust charge fluctuation. The non-adiabatic (τ_{ch}/τ_d is small but finite) dust charge variation, where τ_{ch} is the charging time scale and τ_d is the hydrodynamical time scale, provides an alternate physical mechanism causing the dissipation and waves propagate in this system would be modified significantly. In this paper, we investigate the combined effects of the non-adiabatic dust charge fluctuation, the inhomogeneous equilibrium dust density, and the inhomogeneous equilibrium charging on non-linear properties of the dust acoustic wave in dusty plasmas. It is shown that both solitary waves and shock waves (both oscillatory and monotone shock) can exist in this complex plasma. The variation of non-uniform wave amplitude is given analytically and it indicates that our results are more accurate than that predicated in previous work [12–17]. These are the main results of the present paper.

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2. Governing equation

We consider a three-component collisionless dusty plasmas system consists of micron-sized, negatively charged dust fluid, Boltzmann distributed electrons and ions. Assume the one-dimension problem with spatial weak inhomogeneity along x direction. Because of heavy dust grain, the collision rate of the dust grains with electrons and ions is usually negligible small in the dusty plasma. For simplicity, the collision of dust grains with neutral particles is also neglected, especially, for low neutral particles density case. Suppose in the steady state, the equilibrium dust charging, the equilibrium dust, electron and ion densities vary with x coordinate, i.e., $Z_{d0} = Z_{d0}(x)$ and $n_{(d,i,e)0} = n_{(d,i,e)0}(x)$. When the inhomogeneous is weak, we assume that the equilibrium state is quasi-neutral and that is defined as $n_{i0}(x) = Z_{d0}(x)n_{d0}(x) + n_{e0}(x)$. On slow dust acoustic time scale along with the above assumptions of collisionless and weak inhomogeneity, the phase speed of waves is much smaller than the thermal speeds of ions and electrons, the inertial of ions and electrons can be neglected, and the ions and electrons can be considered as in local thermodynamic equilibrium with their densities obeying the Boltzmann distribution. Hence, under above conditions, the dynamics of the dust acoustic wave can be described by the following equations:

$$\begin{aligned}
&\frac{\partial n_d}{\partial t} + \frac{\partial (n_d u_d)}{\partial x} = 0,\\
&\frac{\partial u_d}{\partial t} + u_d\frac{\partial u_d}{\partial x} = -q\frac{\partial \phi}{\partial x},\\
&\frac{\partial^2 \phi}{\partial x^2} = -qn_d + n_e - n_i,
\end{aligned} \tag{1}$$

where u_d is the dust fluid velocity normalized to the dust acoustic speed $C_d = \sqrt{Z_{d0}(0)T_i/m_d}$ with T_i being the ion-temperature (in energy units) and m_d being the mass of negatively charged dust particles; n_d is the dust grain number density normalized to $n_{d0}(0)$ (dust density at $x = 0$); ϕ is the electrostatic potential normalized to T_i/e with e being the magnitude of the electron charge; the time and space variables are normalized by the dust plasma period $\omega_{pd}^{-1} = \sqrt{m_d/4\pi n_{d0}(0)Z_{d0}^2(0)e^2}$ and the Debye length $\lambda_{Dd} = C_d/\omega_{pd}$, respectively. The charge $q = Z_d(x)e$ (which includes the sign of the charge) is normalized by $Z_{d0}(0)e$ (dust charge at $x = 0$). The Boltzmann distributed electrons and ions in normalized form are

$$n_i = \frac{v(x)}{1-\mu(0)}\exp(-\phi), \tag{2}$$

$$n_e = \frac{\mu(x)}{1-\mu(0)}\exp(\sigma_i\phi), \tag{3}$$

where $\mu(x) = n_{e0}(x)/n_{i0}(0)$, $v(x) = n_{i0}(x)/n_{i0}(0)$, $\sigma_i = T_i/T_e$, with T_e being the electron-temperature (in energy units).

For the case of isolated dust grains, the charge on the dust is easily calculated for spherical grains of radius a. Since the grain is electrically floating, it collects electrons and ions. The floating potential acquired by a charged dust grain in a plasma is analogous to the floating potential of a similar shaped probe in a plasma. In other words, the dust charging mechanisms are mostly based on electrostatic probe theories in plasmas [18]. These theories determine the electron and ion currents reaching by the probes. The currents are termed as orbit limited when the size of the dust grain is smaller than the effective Debye length. In the orbit-limited theory, the currents to the particulates are calculated by assuming that only those electrons and ions are collected whose collisionless orbits intersect the probe's surface. Using this theory, the dust charge q is determined by the following normalized orbital motion-limited [18] charge current balance equation:

$$\frac{\tau_{ch}}{\tau_d}\left(\frac{\partial q}{\partial t} + u_d\frac{\partial q}{\partial x}\right) = \frac{\tau_{ch}}{Z_{d0}(0)e}(I_e + I_i), \tag{4}$$

where I_e and I_i are the electron and ion current, respectively. For weak inhomogeneous and when the streaming velocities of the electrons and ions are much smaller than their corresponding thermal velocities, the normalized expressions for the electron and ion currents for spherical dust grains with radius a are given

$$I_e = -\pi a^2 e\sqrt{\frac{8T_e}{\pi m_e}}n_{e0}(x)\exp(\sigma_i\phi)\exp(zq), \tag{5}$$

$$I_i = \pi a^2 e\sqrt{\frac{8T_i}{\pi m_i}}n_{i0}(x)\left(1 - \frac{zq}{\sigma_i}\right)\exp(-\phi), \tag{6}$$