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Simplified approach to the exponential stability of delayed neural networks with time varying delays

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Abstract

Sufficient conditions in the form of linear matrix inequalities for the exponential stability of the equilibrium point for delayed neural networks with time varying delays are presented. The conditions turn out to be greatly simplified versions of the exponential stability results previously reported by Yucel and Arik. A distinct feature of the present criteria is that they are free of the degree of exponential stability. This feature makes the criteria computationally very attractive.

1. Introduction

The stability properties of neural networks have been studied extensively (see [1–46] and the references cited therein). The present paper deals with the problem of exponential stability of Hopfield-type neural networks with delay, the so-called delayed neural networks (DNNs). In particular, the exponential stability results given in [31] are revisited. The greatly simplified versions of the criteria of [31] are presented. The simplification lies in the fact that it is not necessary to involve $\tau(t)$ and k, where $\tau(t)$ denotes the delay and k the degree of exponential stability. The linear matrix inequality (LMI) feature is retained in the present criteria. Owing to k not being involved, the present criteria turn out to be computationally very attractive.

2. Model descriptions, preliminaries and previous criteria

The DNN model to be considered presently is defined by the following state equations:

$$\frac{\mathrm{d}\boldsymbol{y}(t)}{\mathrm{d}t} = -\boldsymbol{A}\boldsymbol{y}(t) + \boldsymbol{W}_0\boldsymbol{g}(\boldsymbol{y}(t)) + \boldsymbol{W}_1\boldsymbol{g}(\boldsymbol{y}(t-\tau(t))) + \boldsymbol{u}, \tag{1}$$

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or

$$\frac{\mathrm{d}y_i(t)}{\mathrm{d}t} = -a_i y_i(t) + \sum_{j=1}^{n} w_{ij}^0 g_j(y_j(t)) + \sum_{j=1}^{n} w_{ij}^1 g_j(y_j(t-\tau(t))) + u_i, \quad i = 1,2,\ldots,n, \tag{2}$$

where $\boldsymbol{y}(t) = [\,y_1(t) \quad y_2(t) \quad \cdots \quad y_n(t)\,]^{\mathrm{T}}$ is the state vector associated with the neurons, $\boldsymbol{A} = \mathrm{diag}(a_1, a_2, \ldots, a_n)$ is a positive diagonal matrix ($a_i > 0$, $i = 1,2,\ldots,n$), $\boldsymbol{W}_0 = (w_{ij}^0)_{n\times n}$ and $\boldsymbol{W}_1 = (w_{ij}^1)_{n\times n}$ are the connection weight and the delayed connection weight matrices, respectively, $\boldsymbol{u} = [\,u_1 \quad u_2 \quad \cdots \quad u_n\,]^{\mathrm{T}}$ is a constant external input vector, $\tau(t)$ is the transmission delay, the g_j, $j = 1,2,\ldots,n$, are the activation functions, $\boldsymbol{g}(\boldsymbol{y}(\cdot)) = [\,g_1(y_1(\cdot)) \quad g_2(y_2(\cdot)) \quad \cdots \quad g_n(y_n(\cdot))\,]^{\mathrm{T}}$, and the superscript T to any vector (or matrix) denotes the transpose of that vector (or matrix). Throughout this paper, it is understood that $\tau(t)$ is finite for all t. The activation functions are assumed to satisfy the following restrictions:

$$0 \leqslant \frac{g_j(\xi_1) - g_j(\xi_2)}{\xi_1 - \xi_2} \leqslant \sigma_j, \quad j = 1,2,\ldots,n \tag{3}$$

for each $\xi_1, \xi_2 \in R$, $\xi_1 \neq \xi_2$, where σ_j are positive constants.

Let $\boldsymbol{y}^* = [\,y_1^* \quad y_2^* \quad \cdots \quad y_n^*\,]^{\mathrm{T}}$ be an equilibrium point of system (1). The transformation $\boldsymbol{x}(\cdot) = \boldsymbol{y}(\cdot) - \boldsymbol{y}^*$ puts system (1) into the following form:

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = -\boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{W}_0\boldsymbol{f}(\boldsymbol{x}(t)) + \boldsymbol{W}_1\boldsymbol{f}(\boldsymbol{x}(t-\tau(t))), \quad \boldsymbol{x}(t) = \phi(t), \ \ t \in [-\tau(t), 0) \tag{4}$$

or

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -a_i x_i(t) + \sum_{j=1}^{n} w_{ij}^0 f_j(x_j(t)) + \sum_{j=1}^{n} w_{ij}^1 f_j(x_j(t-\tau(t))), \quad i = 1,2,\ldots,n, \tag{5}$$

where $\boldsymbol{x}(t) = [\,x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)\,]^{\mathrm{T}}$ is the state vector of the transformed system, $\boldsymbol{f}(\boldsymbol{x}(\cdot)) = [\,f_1(x_1(\cdot)) \quad f_2(x_2(\cdot)) \quad \cdots \quad f_n(x_n(\cdot))]^{\mathrm{T}}$, and $f_j(x_j(\cdot)) = g_j(x_j(\cdot) + y_j^*) - g_j(y_j^*)$, $j = 1,2,\ldots,n$. Under the restrictions on $g_j(\cdot)$, the functions $f_j(\cdot)$ satisfy the following conditions:

$$0 \leqslant \frac{f_j(\xi_1) - f_j(\xi_2)}{\xi_1 - \xi_2} \leqslant \sigma_j, \quad j = 1,2,\ldots,n \tag{6}$$

for each $\xi_1, \xi_2 \in R$, $\xi_1 \neq \xi_2$.

In the following, $\boldsymbol{B}^{-1}$ denotes the inverse of a square matrix $\boldsymbol{B}$ and the notation $\boldsymbol{B} > \boldsymbol{0}$ ($\boldsymbol{B} \geqslant \boldsymbol{0}$) means that $\boldsymbol{B}$ is symmetric positive definite (positive semidefinite).

Definition 1. Consider the system defined by (4). If there exist a positive constant $k > 0$ and $\gamma(k) > 0$ such that

$$\|\boldsymbol{x}(t)\| \leqslant \gamma(k)\mathrm{e}^{-kt} \sup_{-\tau(t)\leqslant\theta\leqslant 0} \|\boldsymbol{x}(\theta)\|, \quad \forall t > 0, \tag{7}$$

then the origin of (4) is exponentially stable, where k is called the degree of exponential stability.

In [31], the following results are presented:

Theorem 1. *Suppose that in system (4), $\tau(t)$ satisfies $\dot{\tau}(t) \leqslant \eta < 1$. Let $\Sigma = \mathrm{diag}(\sigma_i > 0)$. If the condition (3) is satisfied and there exist positive definite matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$, a positive diagonal matrix $\boldsymbol{D}$, and a positive constant k such that*

$$\boldsymbol{PA} + \boldsymbol{AP} - 2k\boldsymbol{P} - \boldsymbol{P} - 4k\Sigma\boldsymbol{D} - (1-\dot{\tau}(t))^{-1}\mathrm{e}^{2k\tau(t)}\boldsymbol{PW}_1\boldsymbol{Q}^{-1}\boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{P} > \boldsymbol{0}, \tag{8}$$

$$2\boldsymbol{DA}\Sigma^{-1} - \boldsymbol{DW}_0 - \boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{D} - \boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{PW}_0 - 2\boldsymbol{Q} - (1-\dot{\tau}(t))^{-1}\mathrm{e}^{2k\tau(t)}\boldsymbol{DW}_1\boldsymbol{Q}^{-1}\boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{D} \geqslant \boldsymbol{0}, \tag{9}$$

then the origin of system (4) is exponentially stable.

Theorem 2. *Suppose that in system (4), $\tau(t)$ satisfies $\dot{\tau}(t) \leqslant \eta < 1$. Let $\Sigma = \mathrm{diag}(\sigma_i > 0)$. If the condition (3) is satisfied and there exist positive definite matrices $\boldsymbol{P}$ and $\boldsymbol{Q}$, a positive diagonal matrix $\boldsymbol{D}$, and a positive constant k such that*

$$\boldsymbol{PA} + \boldsymbol{AP} - 2k\boldsymbol{P} - \boldsymbol{P} - 4k\Sigma\boldsymbol{D} - 2\boldsymbol{Q} - (1-\dot{\tau}(t))^{-1}\mathrm{e}^{2k\tau(t)}\boldsymbol{PW}_1\Sigma\boldsymbol{Q}^{-1}\Sigma\boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{P} > \boldsymbol{0}, \tag{10}$$

$$2\boldsymbol{DA}\Sigma^{-1} - \boldsymbol{DW}_0 - \boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{D} - \boldsymbol{W}_0^{\mathrm{T}}\boldsymbol{PW}_0 - (1-\dot{\tau}(t))^{-1}\mathrm{e}^{2k\tau(t)}\boldsymbol{DW}_1\Sigma\boldsymbol{Q}^{-1}\Sigma\boldsymbol{W}_1^{\mathrm{T}}\boldsymbol{D} \geqslant \boldsymbol{0}, \tag{11}$$

then the origin of system (4) is exponentially stable.