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Suppression of chaos by weak resonant excitations in a non-linear oscillator with a non-symmetric potential

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Abstract

We examine the Melnikov criterion for transition to chaos in case of one degree of freedom non-linear oscillator with non-symmetric potential. This system, when subjected to an external periodic force, shows homoclinic transition from regular vibrations to chaos just before escape from a potential well. We focus especially on the effect of a second resonant excitation with a different phase on the system transition to chaos. We propose a way of its control.

1. Introduction

In this section we examine vibration and show possible non-feedback control of chaos in a simple, one degree of freedom system subjected to an external excitation with a non-symmetric stiffness given by the following equation:

$$\ddot{x} + \alpha\dot{x} + \delta x + \gamma x^2 = \mu \cos \omega t, \tag{1}$$

where x is a displacement, $\alpha\dot{x}$ is linear damping, $\mu \cos \omega t$ is an external excitation, while δx and γx^2 are linear and quadratic force terms:

$$F(x) = -\delta x - \gamma x^2. \tag{2}$$

Systems, where a quadratic term breaks the reflection symmetry of a potential $V(x)$,

$$V(x) \neq V(-x), \quad \text{and} \quad F(x) = -\frac{\partial V(x)}{\partial x} \tag{3}$$

have been a subject of studies for many years [1–10]. These investigations were motivated by possible applications in description of mostly mechanical systems [1,3,5,7,8] and the catastrophe theory [11]. They were also linked to possible metastable states of atoms and they appeared in problems within the elastic theory [2,12].

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Systems showing homoclinic orbits can be tackled analytically by perturbation methods, namely by the Melnikov method treating [13,14] excitation and damping terms in higher order. Such a treatment has been applied to selected problems with both symmetric and non-symmetric non-linear forces [13,14,10,15] to derive the necessary condition for a transition to chaotic motion via a transversal intersection of homoclinic orbits. Other solutions system parameters regular and chaotic regions can be stabilised by using an additional weak resonant excitation [16–20]. We apply this method to a non-linear system given by Eq. (1). When combined with the Melnikov approach it will predict analytically the range of parameters taming the chaotic behaviour. This document is divided into five sections. The introduction is followed by Section 2 which deals with Melnikov analysis. Next section is devoted to an analysis of a weak resonant excitation term and its useful role in system control. In Section 4, the analytic predictions are confirmed by means of numerical simulations. Section 5 concludes our results.

2. Melnikov analysis

This section follows the discussion initiated by Thompson [2] where he derived the analytic formula for a critical amplitude of a non-linear oscillator as described by an equation similar to our Eq. (1). We decided to include this section as an important preface part to our main discussion to be given in the next section.

Thus we start our analysis from the second order equation of motion (Eq. (1)). We obtain

$$\begin{aligned} \dot{x} &= v, \\ \dot{v} &= -\delta x - \gamma x^2 + \epsilon[-\tilde{\alpha} v + \tilde{\mu}\cos\omega t] \end{aligned} \tag{4}$$

by transforming the second order equation of motion (Eq. (1)) into two differential equations of the first order. Looking for stable and unstable manifolds we introduce a small parameter ϵ to the above equations and renormalise parameters $\tilde{\alpha}$ and $\tilde{\mu}$ via $\alpha = \epsilon\tilde{\alpha}$ and $\mu = \epsilon\tilde{\mu}$, respectively.

The corresponding unperturbed Hamiltonian have the following form:

$$H^0 = \frac{v^2}{2} + V(x), \tag{5}$$

where

$$V(x) = \frac{\delta x^2}{2} + \frac{\gamma x^3}{3} \tag{6}$$

is a potential of a non-symmetric well plotted in Fig. 1a. This function has the local peak at the point

$$x_0 = -\frac{\delta}{\gamma}. \tag{7}$$

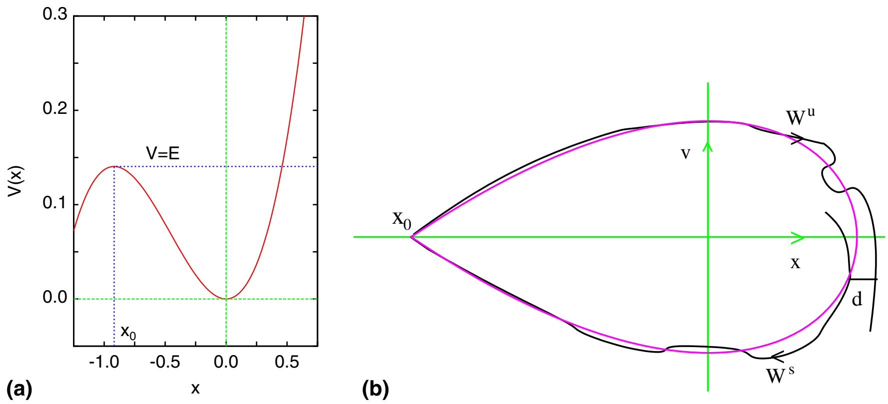

Fig. 1. (a) The potential well of the unperturbed Hamiltonian (Eq. (5)) for $\delta = 1$ and $\gamma = 1.089$, the energy level $V = E$ corresponds to the Hamiltonian system with a tangent point at x_0; (b) stable W^S and unstable W^U manifolds for unperturbed (gray) and a damped and excited system (in black). The distance d between W^S and W^U can be described by Melnikov function $M(t_0)$ (Eq. (9)). *Note*: x_0 indicates the local extremum of potential $V(x)$ (a) which simultaneously represents a saddle point in the phase plane (b).