Available online at www.sciencedirect.com
ScienceDirect
Chaos, Solitons and Fractals 32 (2007) 832–846
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

Adaptive regulation and set-point tracking of the Lorenz attractor

H.N. Pishkenari [a], M. Shahrokhi [b,*], S.H. Mahboobi [a]

[a] *Department of Mechanical Engineering, Sharif University of Technology, P.O. Box 11365-9465, Tehran, Iran*
[b] *Department of Chemistry and Petroleum Engineering, Sharif University of Technology, P.O. Box 11365-9465, Tehran, Iran*

Accepted 10 November 2005

Communicated by Prof. M.S. El Naschie

Abstract

In this paper, an approach is proposed for controlling the uncertain Lorenz system. Based on an identification technique, a controller is designed that guarantees the regulation of all states in the presence of system uncertainty. Since in some applications the challenging problem of output tracking is desired, we have proposed several effective set-point tracking control techniques. The control schemes that are based on the feedback linearization method, can stabilize the internal dynamics of the system. Simulation results have illustrated the effectiveness of the proposed schemes.

1. Introduction

Control of the Lorenz system [1] has been studied extensively by several researchers in recent years [2]. The Lorenz system is a simplified model of a thermally driven fluid convection system between parallel plates. Depending on the system parameters, such a system exhibits a rich spectrum of responses. Since the chaotic systems are nonlinear, the methods based on linear state feedback, usually cannot guarantee the system stability in the global sense. To obtain the global stability, many analysis and synthesis methods for nonlinear control systems have been proposed, such as backstepping control [3–5], sliding mode control [6], feedback linearization control [7], active control [8], etc. Gallegos [9] applied the input-state feedback linearization to control the Lorenz system. It has been discovered that the complete input-state linearization results in controller singularity when applied to the Lorenz system [10]. Regarding the singularity problem in controlling the Lorenz system, Lenz and Obradovic [10] and Zeng and Singh [11] have obtained some singularity-free results and proposed a global approach using partial linearization. Yu [12] proposed a variable structure control strategy to stabilize the Lorenz chaos. To prevent an exploding control action due to singular control law, lower and upper bounds have been considered for the control action and consequently the resulting controller is only

* Corresponding author. Tel.: +98 21 66165419; fax: +98 21 66022853.
E-mail address: shahrokhi@sharif.edu (M. Shahrokhi).

0960-0779/$ - see front matter
doi:10.1016/j.chaos.2005.11.054

locally stabilizing. Vincent and Yu [13] proposed a bang–bang optimal control for stabilizing the unstable equilibrium points of the system. Gao et al. [7] studied the nonlinear feedback control of the Lorenz system based on state space design. Chen and Liu [14] investigated the nonlinear regulation of the Lorenz system by using the feedback linearization techniques with different control structures and control objectives. A control method for the Lorenz system combining local and global techniques is also proposed in [15].

Some researchers [16–18] have proposed adaptive control schemes for controlling the chaotic systems. Liu et al. [19] proposed an adaptive control scheme for controlling a partially unknown unified system. By applying an observer for identification of the unknown parameter, via extending equilibrium manifolds of the original system, a simple controller is designed. It should be noted that in most of the conducted researches, the complete or partial knowledge of system parameters are assumed.

In this paper, a feedback linearization control approach is proposed for controlling the Lorenz chaotic system. The control objectives are: regulation and output tracking. The proposed controller for regulation guarantees the regulation of all states. For set-point tracking, three different controllers have been designed depending on the state that should be tracked. To extend the controllers for the system with unknown parameters, an identifier has been designed, and used in the control structure.

2. A general identification method

Consider the general nonlinear system given below:

$$\dot{x}_i = \mathbf{a}_i \mathbf{f}_i(\mathbf{x}, u) + g_i(\mathbf{x}, u), \quad 1 \leqslant i \leqslant n \tag{1}$$

where

$$\mathbf{x}^{\mathrm{T}} = (x_1, x_2, \ldots, x_n)$$
$$\mathbf{a}_i^{\mathrm{T}} = (a_{i1}, a_{i2}, \ldots, a_{im_i})$$
$$\mathbf{f}_i^{\mathrm{T}}(\mathbf{x}, u) = (f_{i1}(\mathbf{x}, u), f_{i2}(\mathbf{x}, u), \ldots, f_{im_i}(\mathbf{x}, u))$$

$\mathbf{x}$ is the vector of state variables and u is the plant input. Assume that vectors $\mathbf{a}_i$, $1 \leqslant i \leqslant n$ are unknown, while the nonlinear vector functions $\mathbf{f}_i(\mathbf{x}, u)$ and $\mathbf{g}_i(\mathbf{x}, u)$ are known. The objective is identification of the unknown parameters a_{ij} $(1 \leqslant i \leqslant n, 1 \leqslant j \leqslant m_i)$. Since the number of unknown parameters in each equation (each row) is m_i, the total number of unknown parameters will be $\sum_{i=1}^{n} m_i$. It is assumed that all of the state variables are available.

Consider the system model as given below:

$$z_i = \hat{\mathbf{a}}_i^{\mathrm{T}} \frac{\bar{\mathbf{f}}_i}{s + k_i} + \frac{\bar{g}_i}{s + k_i} + \frac{k_i x_i(s)}{s + k_i} + \alpha_i \tag{2}$$

where

$$\bar{\mathbf{f}}_i = L(\mathbf{f}_i(\mathbf{x}, u)) \tag{3}$$
$$\bar{g}_i = L(g_i(\mathbf{x}, u)) \tag{4}$$
$$\alpha_i(s) = L(x_i(0) \mathrm{e}^{-k_i t}) \tag{5}$$

In the above equations, L is the Laplace transform operator, vector $\hat{\mathbf{a}}_i$ is the estimation of $\mathbf{a}_i$ and $k_i > 0$ is a tuning parameter. The error is defined below:

$$e_i = z_i - x_i \quad \text{or} \quad \mathbf{e} = \mathbf{z} - \mathbf{x} \tag{6}$$

Using Eqs. (1) and (2) and performing some manipulations yields:

$$e_i = (\hat{\mathbf{a}}_i - \mathbf{a}_i)^{\mathrm{T}} \frac{\bar{\mathbf{f}}_i}{s + k_i} \tag{7}$$

2.1. Gradient method

A gradient-based identifier is proposed and its stability can be shown using the Lyapunov stability theorem. The error can be written as below:

$$e_i = \boldsymbol{\varphi}_i^{\mathrm{T}} \boldsymbol{\delta}_i = \boldsymbol{\delta}_i^{\mathrm{T}} \boldsymbol{\varphi}_i \tag{8}$$