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The periodic wave solutions for the (2 + 1)-dimensional dispersive long water equations

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Abstract

Periodic wave solutions expressed by Jacobi elliptic functions for the (2 + 1)-dimensional dispersive long water equations are obtained by using the extended *F*-expansion method. In the limit cases, the solitary wave solutions and the trigonometric function solutions for the equations are also obtained.

1. Introduction

Since the soliton phenomena were first observed by Scott Russell in 1834 [1] and the KdV equation was solved by the inverse scattering method by Gardner et al. in 1967 [2], finding explicit solutions of nonlinear evolution equations (NEEs) has become one of the most exciting and extremely active areas of research investigation. Various methods for obtaining explicit solutions of NEEs have been proposed. Among these are Hirota's bilinear method [3], Bäcklund transformation [4], Painlevé expansion [5], tanh method [6–9], sine–cosine method [10], homotopy perturbation method [11–13], adomian method [14], homogenous balance method [15], variational approach [16–19], algebraic method [20], Jacobi elliptic function expansion method [21], *F*-expansion method [22–24], and so on. *F*-expansion method is an overall generalization of Jacobi elliptic function expansion method, it provides an effective method to obtain periodic wave solutions of a lot of NEEs.

For the (2 + 1)-dimensional dispersive long water equations

$$u_{yt} + H_{xx} + \frac{1}{2}(u^2)_{xy} = 0, \tag{1.1}$$

$$H_t + (uH + u + u_{xy})_x = 0, \tag{1.2}$$

multiple soliton-like solutions, Jacobi elliptic periodic solutions and other exact solutions have been obtained in Refs. [24–27]. In this paper, we will apply the extended *F*-expansion method [23] in which the inverse powers of $F(\xi)$ are involved to Eqs. (1.1) and (1.2). As a result, new periodic wave solutions expressed by Jacobi elliptic functions are obtained.

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2. The concentrated formulas of traveling wave solutions

Using the transformation $u(x,y,t)=u(\xi)$, $H(x,y,t)=H(\xi)$, $\xi=kx+ly+wt$, we reduce Eqs. (1.1) and (1.2) to ODEs

$$wlu''+k^2H''+kluu''+klu'^2=0, \tag{2.1}$$

$$wH'+ku'H+kuH'+ku'+k^2lu'''=0, \tag{2.2}$$

where $u'=\mathrm{d}u(\xi)/\mathrm{d}\xi$, $u''=\mathrm{d}^2u(\xi)/\mathrm{d}\xi^2$, $u'''=\mathrm{d}^3u(\xi)/\mathrm{d}\xi^3$, $H'=\mathrm{d}H(\xi)/\mathrm{d}\xi$, $H''=\mathrm{d}^2H(\xi)/\mathrm{d}\xi^2$.

Supposing that the solutions of ODEs (2.1) and (2.2) can be expressed by

$$u(\xi)=\sum_{i=-N}^{N}a_iF^i(\xi),\quad H(\xi)=\sum_{j=-L}^{L}b_jF^j(\xi), \tag{3}$$

where $a_0, a_{\pm1},\ldots,a_{\pm N}, b_0, b_{\pm1},\ldots,b_{\pm L}$ are constants to be determined later, and a_N, $b_L\neq 0$. $F(\xi)$ satisfies the first order nonlinear ODE

$$F'^2(\xi)=PF^4(\xi)+QF^2(\xi)+R, \tag{4}$$

and hence holds for $F(\xi)$

$$F''(\xi)=2PF^3(\xi)+QF(\xi), \tag{5}$$

where P, Q, R are constants.

According to the homogenous balance method [15], we can get $N=1$, $L=2$, so we have

$$u(\xi)=a_{-1}F^{-1}(\xi)+a_0+a_1F(\xi), \tag{6.1}$$

$$H(\xi)=b_{-2}F^{-2}(\xi)+b_{-1}F^{-1}(\xi)+b_0+b_1F(\xi)+b_2F^2(\xi). \tag{6.2}$$

Substituting (6.1) and (6.2) into ODEs (2.1) and (2.2), using (4) and (5), the left-hand sides of ODEs (2.1) and (2.2) are converted into finite power series of $F(\xi)$, setting each coefficient of powers of $F(\xi)$ to zeros, yields a set of algebraic equations for a_{-1}, a_0, a_1, b_{-2}, b_{-1}, b_0, b_1, b_2, k, l and w as follows:

$$F^{-4}(\xi): 3a_{-1}^2klR+6b_{-2}k^2R=0,$$
$$F^{-3}(\xi): 2a_{-1}a_0klR+2b_{-1}k^2R+2a_{-1}wlR=0,$$
$$F^{-2}(\xi): 2a_{-1}^2klQ+4b_{-2}k^2Q=0,$$
$$F^{-1}(\xi): a_{-1}a_0klQ+b_{-1}k^2Q+a_1wlQ=0,$$
$$F^{0}(\xi): a_{-1}^2klP+2b_{-2}k^2P+a_1^2klR+2b_2k^2R=0,$$
$$F^{1}(\xi): a_0a_1klQ+b_1k^2Q+a_1wlQ=0,$$
$$F^{2}(\xi): 2a_1^2klQ+4b_2k^2Q=0,$$
$$F^{3}(\xi): 2a_0a_1klP+2b_1k^2P+2a_1wlP=0,$$
$$F^{4}(\xi): 3a_1^2klP+6b_2k^2P=0,$$
$$F^{-4}(\xi): -3a_{-1}b_{-2}k-6a_{-1}k^2lR=0,$$
$$F^{-3}(\xi): -2a_0b_{-2}k-2a_{-1}b_{-1}k-2b_{-2}w=0,$$
$$F^{-2}(\xi): -a_{-1}k-a_{-1}b_0k-a_1b_{-2}k-a_0b_{-1}k-a_{-1}k^2lQ-b_{-1}w=0,$$
$$F^{0}(\xi): a_1k+a_1b_0k+a_0b_1k+a_{-1}b_2k-6a_{-1}k^2l+6a_{-1}k^2lP+a_1k^2lQ+b_1w=0,$$
$$F^{1}(\xi): 2a_1b_1k+2a_0b_2k+2b_2w=0,$$
$$F^{2}(\xi): 3a_1b_2k+6a_1k^2l=0.$$

Solving above algebraic equations, we get two kinds of solutions and only list the solutions under the condition of $a_{-1}\neq 0$, $a_1\neq 0$, $b_{-2}\neq 0$ and $b_2\neq 0$ as following

I. when $P=1$, $R>0$, $k\neq 0$ and $l\neq 0$

$$a_{-1}=2k\sqrt{R},\quad a_0=-\frac{w}{k},\quad a_1=\pm 2k, \tag{7.1}$$

$$b_{-2}=-2klR,\quad b_{-1}=0,\quad b_0=-1-klQ\pm 2kl\sqrt{R},\quad b_1=0,\quad b_2=-2kl, \tag{7.2}$$