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Recent advances on failure and recovery in networks of networks

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ABSTRACT

Until recently, network science has focused on the properties of single isolated networks that do not interact or depend on other networks. However it has now been recognized that many real-networks, such as power grids, transportation systems, and communication infrastructures interact and depend on other networks. Here, we will present a review of the framework developed in recent years for studying the vulnerability and recovery of networks composed of interdependent networks. In interdependent networks, when nodes in one network fail, they cause dependent nodes in other networks to also fail. This is also the case when some nodes, like for example certain people, play a role in two networks, i.e. in a multiplex. Dependency relations may act recursively and can lead to cascades of failures concluding in sudden fragmentation of the system. We review the analytical solutions for the critical threshold and the giant component of a network of n interdependent networks. The general theory and behavior of interdependent networks has many novel features that are not present in classical network theory. Interdependent networks embedded in space are significantly more vulnerable compared to non-embedded networks. In particular, small localized attacks may lead to cascading failures and catastrophic consequences. Finally, when recovery of components is possible, global spontaneous recovery of the networks and hysteresis phenomena occur. The theory developed for this process points to an optimal repairing strategy for a network of networks. Understanding realistic effects present in networks of networks is required in order to move towards determining system vulnerability.

1. Introduction

Classical graph studies involved simple random graphs (Erdős–Rényi networks) or regular lattices, however once more data became available about real-world complex systems, researchers quickly discovered that real networks have far more complex structures. First of all, many real networks have some nodes that act as hubs with far more connections than other nodes [1–3]. Beyond this, many studies have found other non-random structures such as the small-world structure [4], community structure [5,6], clustering [4], degree–degree correlations [7,8], and unique spatial structures [9] in networks. Understanding the topological structure of real-world networks has provided insights into fields as diverse as epidemiology [10–14], climate [15,16], economics [17,18], sociology [19], infrastructure [20], traffic [21], physiological networks [22], and brain networks [23,24].

One of the most important properties of networks is their robustness to failures or in other words what fraction of nodes remain connected after some other subset

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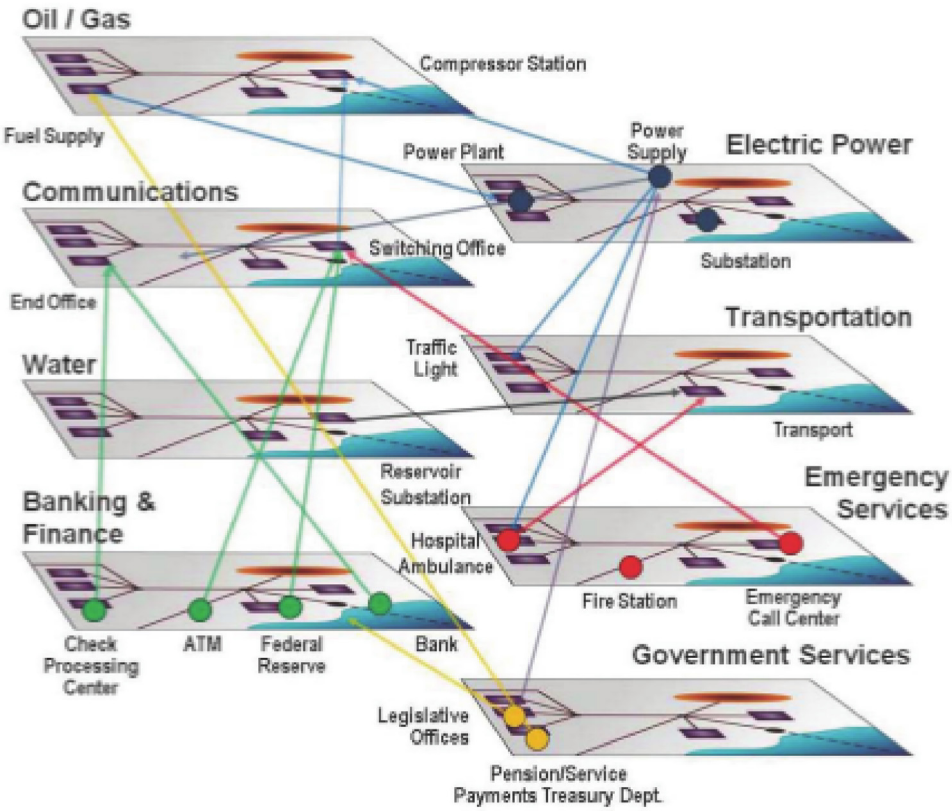

Fig. 1. Modern infrastructure involves many dependency relations as shown in the figure from [44].

of nodes is removed? To answer this question it is often useful to use percolation theory which tells us what fraction of nodes are in the largest component, P_∞ [25,26]. Defining functionality based on the size of the largest component makes sense in many contexts. For example, in the case of a communications network it is usually pertinent to ask what fraction of nodes are able to communicate? If $P_\infty \approx 1$ then the network is functional and most nodes can easily communicate. However if $P_\infty \approx 0$ then very few nodes can communicate and the network is essentially non-functional. The term *giant connected component* is used when P_∞ is a non-zero fraction of an infinite system.

The formal framework of percolation theory in the context of networks involves varying $1-p$, the fraction of nodes removed at random and calculating the corresponding size of the largest component, $P_\infty(p)$. In general, for single isolated networks $P_\infty(p)$ undergoes a second-order, continuous phase transition [27] as p decreases. The point where the transition occurs is typically referred to as p_c. For $p > p_c$ we have $P_\infty(p) > 0$, but for $p < p_c$, $P_\infty(p) = 0$. For Erdős–Rényi networks it was found that $p_c = 1/\langle k\rangle$ where $\langle k\rangle$ is the mean degree of the network [28–30]. In contrast, for scale-free networks where the degree distribution follows $p(k) \sim k^{-\lambda}$, it was found that for $\lambda < 3$, $p_c = 0$ [31], indicating that only when essentially all nodes are removed does the giant component reach zero.

Most real networks do not operate in isolation, but are instead merely one system in a network of networks [32–38]. One type of important relationship between networks is interdependence [39,40]. This occurs in infrastructure where power grids may depend on communications systems and in many biological systems where functionality requires numerous organs and metabolic pathways to work together. Another example occurs in sociology where an individual may participate in multiple social networks [41–43]. We show an example of the complexity of interdependence in modern infrastructure in Fig. 1. Here, we will review some of our recent results on interdependent networks and point readers to other articles where they can learn more about the subject.

In interdependent networks there are two types of links, the usual connectivity links that are also present in single networks, as well as a new type of links called *dependency links* [39,40,45–49]. These dependency links imply that the node at one end of the link relies on the node at the other end of the link to function. Thus if the node on one end of a dependency link fails, then the node on the other end will also fail.

The structure of the network of networks is based on networks having dependency links between them. Possible structures are shown in Fig. 2 and include tree-like structures, a loop, and a random-regular configuration where each networks has the same number of dependency relations.