Chaos, Solitons and Fractals 90 (2016) 46–54

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Can topology reshape segregation patterns?

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ARTICLE INFO

Article history:
Received 1 November 2015
Revised 18 February 2016
Accepted 19 February 2016
Available online 12 March 2016

Keywords:
Social dynamics
Schelling model
Segregation
Metapopulation
Coarsening process

ABSTRACT

We consider a metapopulation version of the Schelling model of segregation over several complex networks and lattices. We show that the segregation process is topology independent and hence it is intrinsic to the individual tolerance. The role of the topology is to fix the places where the segregation patterns emerge. In addition we address the question of the time evolution of the segregation clusters, resulting from different dynamical regimes of a coarsening process, as a function of the tolerance parameter. We show that the underlying topology may alter the early stage of the coarsening process, once large values of the tolerance are used, while for lower ones a different mechanism is at work and it results to be topology independent.

1. Introduction

People tend to move to reduce their uneasiness and increase their personal utility computed across many dimensions, thus the formation of strongly separated groups or even ghettos is a self-organised emergent process of our societies. Such phenomenon can be harmful when leading to discrimination based on segregation [1]. This issue has been recognised as one of the most important socio-political problems in the USA, and many Western-European countries are becoming increasingly aware of this problem [2]. Economical inequalities have been recognised as the main cause for this phenomenon [3]. Not downplaying this fact, Schelling studied, in the 70's, the effect of individual preferences concerning the composition of the individuals neighbourhood. Introducing a stylised model, he showed that even weak preferences unbalance can lead to total segregation in societies [5]. In the modern language of complex systems, such phenomenon can be ascribed to self-organisation in social sciences [6–8] and thus possibly studied with methods coming from statistical mechanics [9].

In the original works by Schelling, a population composed by two kinds of agents, say Red and Blue, is assumed to live on a regular network (1D or 2D lattices) where each node can either host a single agent or remain empty [4,5]. Agents are happy, and thus do not move, if they are surrounded by sufficiently many agents of the same kind, otherwise they are unhappy and move to another location. The model is specified though two parameters: the population density of agents on the lattice, ρ, and the tolerance, ε, the parameter responsible for the willingness to move or to stay in a certain site. Besides its simplicity the model exhibits a very rich phenomenology, for instance scholars have been interested in understanding the presence of a phase transitions in the final distribution of agents (segregation vs well mixed), the role of the density of agents and thus of the density of empty spaces (emptiness), and the dynamics of the separation of clusters composed by the same kind of agents (interface growth) [10,11].

The Schelling model has been used to explain racial segregation phenomena, mainly in the U.S., and despite the simplicity of the assumptions (only two kinds of agents are allowed to move in a square lattice) and its stylised

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http://dx.doi.org/10.1016/j.chaos.2016.02.024
0960-0779/

description of the human behaviour (the tolerances are uniform, symmetric and independent on the number of involved individuals) it is able to reproduce the unstable nature of non-segregated societies [12].

Recently the Schelling model has been improved to account for the possibility that nodes host more than one agent [7,8,13], opening in this way to a generalisation to the metapopulation framework [6]. In particular, in a previous work [13], we studied a metapopulation Schelling model over regular networks (1D and 2D lattices) able to capture the idealised movement of people across city districts, each one allowing for a finite carrying capacity. A first result of this paper is that a phase transition happens when the tolerance is $\varepsilon_c = 0.5$. For $\varepsilon < \varepsilon_c$ the lattice sites are completely magnetised, namely only one type of agents is present on each node. Moreover, we found that for low values of the tolerance ($\varepsilon < 0.3$) the system remains stuck in a long quasi-stationary transient phase, during which the population is in a well mixed phase, but suddenly the system jumps in a new stable phase where agents are heterogeneously distributed across nodes. We named towers, such nodes with a very large density, corresponding thus to nodes where agents prefer to live. Let us observe that such result is obtained without any exogenous preferential attachment mechanism. Another interesting characteristics exhibited by the system, in the range of small tolerances, is the presence of clusters of empty nodes separating monocoloured clusters that we named interface; moreover such interfaces grow in time with a power law behaviour whose specific exponent depends on the tolerance parameter.

In the own Schelling words, people get separated along many lines and many ways, but the pattern formation regarding this separation, has strong connection with how different cultures define their satisfaction, in terms of their daily activity and living constraints. For instance, for families whose children play outside or love having their daily commercial life surrounding their houses, their neighbourhood is very important. On the other hand, if we consider cities, where people activity is largely restricted to the buildings where they live or work, then the individuals relocation decisions depend more on the composition of the fully-connected building than on the neighbourhood.

Building on the previous remark we are interested in the present work to consider a metapopulation Schelling model where the underlaying network exhibits a complex topology, and thus to understand how such heterogeneity of connectivity changes the model outcomes with respect to the regular lattice previously studied.

We find that neither a complex network topology nor the network size do not change drastically the qualitative system behaviour and that the transition threshold leading to segregated states is $\varepsilon = 0.5$, and thus results to be an universal parameter of the Schelling models. This outcome is interesting since it mirrors a previous important result for opinion dynamics, [14], where it has been showed that in Bounded Confidence Models, where a threshold determines if people locally agree or not, the topology is not affecting the final outcome of the system, unless the network itself has a dynamics [15,16].

Even if the qualitative behaviour of the Schelling model is not affected by the underlaying network topology, we observe that the times to the convergent state depends strongly on the connectivity of the network, the larger the average degree the longer the time needed to reach the final state, and scales super linearly with the network size. Moreover we observe important correlations between the local network structure and the population distributions, namely agents tend to avoid hubs, that thus will be mainly empty, while they tend to accumulate, namely to form towers, in low degree nodes.

On the other hand, the removal and rewiring of local connections modifies the typical early coarsening process of clusters growth, for big values of ε. However for low values of tolerance, when the system passes through a long quasi-stationary regime, a late stage of coarsening becomes dominant, where the system creates the "towers". We found that in this regime the time scaling of the interface growth is independent of the network topology. Our work is organised as follow: in Section 2 we provide a detailed description of the methodology in terms of dynamics and topology. The results are shown in Section 3. Finally we conclude in Section 4.

2. The model

Let us consider a network composed by N nodes each one able to contain at most L agents, some of which belong to the Red group and some to the Blue one. The total number of agents is a conserved parameter that we decide to express in terms of the total emptiness, ρNL, where $\rho \in (0, 1)$, is defined as the total number of available space not occupied by any agent. The system is initialised by choosing uniformly random amounts of Blue and Red agents in each node, close to the homogeneous equilibrium $(1-\rho)NL/2$. Time increases by discrete steps. One step consists in the random selection and the (eventual) relocation of an agent accordingly to her happiness. An agent is happy in a given node i, if the fraction of agents with the opposite kind inside the neighbourhood, defined as the set of nodes at distance smaller or equal to 1 from i, is smaller than a threshold ε, in formula:

$$f_i^B = \frac{\sum_{j\in i} n_j^A}{\sum_{j\in i}(n_j^A + n_j^B)} \text{ then } B \text{ is unhappy in node } i, \quad \text{if } f_i^B > \varepsilon\,, \tag{1}$$

where n_i^X, $X = A, B$, denotes the number of agents belonging to the X–kind in the node ith, and $j \in i$ is a shorthand to denote the set of nodes connected to i and i itself. Unhappy agents relocate themselves to another node in the network, chosen with uniformly random probability, and provided that there is enough space available there. The rationale being that one agent cannot have a full knowledge of the arriving site.

In the following, except when explicitly written, we consider networks formed by $N = 400$ nodes and we will consider four topologies: Lattices, Small World networks constructed using the Watts–Strogatz (WS) algorithm [18], Random Networks by using Erdős–Rényi procedure [19] and the Scale-free topologies obtained using the preferential attachment mechanism proposed by Barabasi–Albert (BA) [21]. To check the impact of the node degree we also