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Short Communication

Vacuum-to-vacuum transition probability and radiation in a medium

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HIGHLIGHTS

- Quantum viewpoint of radiation in a medium based on the vacuum-to-vacuum transition probabilities.
- Mathematical method in handling radiation in a medium for arbitrary sources.
- Radiated energy and power for arbitrary current distributions in a medium.
- Explicit power of radiation in a medium in a bounded region.

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ABSTRACT

We recast the vacuum-to-vacuum transition probability for the description of radiation in an isotropic medium of permeability μ, and permittivity ε, in a form which brings us in contact with radiation theory in vacuum. Using the inherited property of such a system, with arbitrary current distributions, of emitting photons via the Poisson distribution, the average number of photons emitted in such a medium is directly obtained from which the power of radiation is readily extracted. As an application, the power of radiation, emitted by a charged particle, in a medium trapped between perfectly conducting neutral parallel plates for arbitrary finite separations is explicitly obtained.

1. Introduction

We consider a homogeneous and isotropic medium of permeability μ and permittivity ε. The Minkowski metric used in this work is defined by $\eta_{\mu\nu} = \text{diag}[-1, 1, 1, 1]$. To describe photons in such a medium, one simply scales $F_{0i}F^{0i} \to \varepsilon F_{0i}F^{0i}$, and $F_{ij}F^{ij} \to F_{ij}F^{ij}/\mu$, in the Lagrangian density, where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. That is, the Lagrangian density becomes $(i, j = 1, 2, 3)$

$$\mathcal{L} = -\frac{1}{4\mu}F_{ij}(x)F^{ij}(x) - \frac{\varepsilon}{2}F_{0i}(x)F^{0i}(x) + J^\mu(x)A_\mu(x). \quad (1)$$

Note that the scaling factors are not ε^2, $1/\mu^2$, respectively, as one may naïvely expect. The reason is that the variations of the action, with respect to the vector potential, involving the quadratic terms $\varepsilon F_{0i}F^{0i}$, $F_{ij}F^{ij}/\mu$, generate the linear terms corresponding to the electric and magnetic field components which are just needed in deriving Maxwell's equations.

We recast the theory in a form which brings us into contact with our earlier treatment (Manoukian, 2015) dealing with radiation in vacuum, from which radiation from an arbitrary current distribution in a medium may be considered in a very general way. We recapitulate the method of study via the vacuum-to-vacuum transition probability in describing radiation. To this end, note that prior to switching on of the current, as a source of photon production, one is dealing with a vacuum state, denoted by $|0_-\rangle$, involving no photons. After switching on of the current, the state of the system may evolve to one involving any number of photons, or it may just stay in the vacuum state, involving no photons, with the latter state now denoted by $|0_+\rangle$. Quantum theory tells us that the vacuum-to-vacuum transition probability satisfies the inequality $|\langle 0_+|0_-\rangle|^2 < 1$, due to conservation of probability, allowing the possibility that the system may evolve to other states as well involving an arbitrary number of photons that may be created by the current source. A very interesting property of this system is that the probability distribution of the photon number N created by the current (Schwinger, 1970) is given by the Poisson distribution (Manoukian, 2011). That is

$$\text{Prob}[N = n] = \frac{(\lambda)^n}{n!}e^{-\lambda}, \quad n = 0, 1, \ldots, \quad (2)$$

$$\lambda = \langle N \rangle, \quad (3)$$

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where $\lambda = \langle N \rangle$ denotes the average number of photons created by the current source, and

$$\exp[-\langle N\rangle] = |\langle 0_+|0_-\rangle|^2, \quad (4)$$

denotes the probability that no photons are created by the current source, i.e., it represents the vacuum-to-vacuum transition probability $|\langle 0_+|0_-\rangle|^2$ as just stated.

Quantum viewpoint analysis of electromagnetic phenomena and electromagnetic radiation, and of related applications, e.g., Manoukian (1991, 1997, 2013, 2015), Manoukian and Charuchittapan (2000), Manoukian and Viriyasrisuwattana (2006), Feynman (1985), Bialynicki-Birula (1996), Kennedy et al. (1980), Deitsch and Candelas (1979), and Schwinger et al. (1976), turns out to be quite useful in applications and certainly in simplifying, to a large extent, derivations in this field.

The derivation given below is quite general and applies to arbitrary current distributions, and is expected to be of interest in other applications and, in particular, in media involving obstacles and in periodic configuration (Bellucci and Maisheev, 2006) and radiation from varying sources (e.g., Budko, 2009; Gal'tsov et al., 2007; Bessonov, 2006; Manoukian, 1991), in general, as well as for further direct generalizations involving quantum corrections. These and other directions of research mentioned below will be attempted in future work.

In Section 2 a general expression is derived, via the vacuum-to-vacuum transition probability, for the average number of photons emitted from an arbitrary current distributions from which the power of radiation from such currents can be readily extracted. For completeness, and for the convenience of the reader, a direct derivation of the classic Čerenkov power of radiation, in infinite extended media, is derived in Section 3 as a preparation for handling more complicated situations. In Section 4, such radiation is considered in a bounded medium consisting of a slab confined between two parallel conducting neutral plates, separated by a finite distance, and an explicit expression for the power of radiation is derived showing the power of the formalism.

2. Average number of photons emitted

We carry out the following scalings in the action involving the Lagrangian density in (1):

$$x^0 = \sqrt{\mu\varepsilon}\,\underline{x}^0, \quad \mathbf{x} = \underline{\mathbf{x}}, \quad (5)$$

$$\partial_0 = \frac{1}{\sqrt{\mu\varepsilon}}\underline{\partial}_0, \quad \partial_i = \underline{\partial}_i, \quad (6)$$

$$A^0(x) = \frac{1}{\mu^{1/4}\varepsilon^{3/4}}\underline{A}^0(x), \quad (7)$$

$$\mathbf{A}(x) = \frac{\mu^{1/4}}{\varepsilon^{1/4}}\underline{\mathbf{A}}(x), \quad (8)$$

$$J^0(x) = \frac{\varepsilon^{1/4}}{\mu^{1/4}}\underline{J}^0(x), \quad (9)$$

$$\mathbf{J}(x) = \frac{1}{\mu^{3/4}\varepsilon^{1/4}}\underline{\mathbf{J}}(x). \quad (10)$$

The action integral, up to an overall factor, thus takes the form

$$W = \int(\mathrm{d}\underline{x})\Big[-\frac{1}{4}\big(\underline{\partial}_\mu\underline{A}_\nu(x) - \underline{\partial}_\nu\underline{A}_\mu(x)\big)\big(\underline{\partial}^\mu\underline{A}^\nu(x) - \underline{\partial}^\nu\underline{A}^\mu(x)\big) + \underline{J}^\mu(x)\underline{A}_\mu(x)\Big]. \quad (11)$$

Note that the argument of $\underline{A}^\mu(x)$ is x and not $\underline{x}$, also that

$$\underline{\partial}_\mu\underline{J}^\mu(x) = \mu^{3/4}\varepsilon^{1/4}\partial_\mu J^\mu(x). \quad (12)$$

The vacuum-to-vacuum transition amplitude is then simply

$$\langle 0_+|0_-\rangle = \exp\Big[\frac{\mathrm{i}}{2\hbar c^3}\int(\mathrm{d}\underline{x})\,(\mathrm{d}\underline{x}')\underline{J}_\mu(x)D_+^{\mu\nu}(\underline{x},\underline{x}')\underline{J}_\nu(x')\Big], \quad (13)$$

as inferred from theory formulated in vacuum, e.g., Manoukian (2015), where $D^{\mu\nu}(x, x')$ is the photon propagator determined below for the cases considered.

The vacuum-to-vacuum transition probability then follows directly from (13) to be

$$|\langle 0_+|0_-\rangle|^2 = \exp\Big[-\frac{1}{\hbar c^3}\int(\mathrm{d}\underline{x})\,(\mathrm{d}\underline{x}')\underline{J}_\mu(x)\big(\mathrm{Im}D_+^{\mu\nu}(\underline{x},\underline{x}')\big)\underline{J}_\nu(x')\Big], \quad (14)$$

where $(\mathrm{d}x) \equiv \mathrm{d}x^0\,\mathrm{d}x^1\,\mathrm{d}x^2\,\mathrm{d}x^3$, from which the average number of photons emitted by the arbitrary current distribution is given by

$$\langle N\rangle = \frac{1}{\hbar c^3}\int(\mathrm{d}\underline{x})\,(\mathrm{d}\underline{x}')\underline{J}_\mu(x)\big(\mathrm{Im}D_+^{\mu\nu}(\underline{x},\underline{x}')\big)\underline{J}_\nu(x'). \quad (15)$$

The latter may be more conveniently rewritten as

$$\langle N\rangle = \frac{1}{\mu\varepsilon\hbar c^3}\int(\mathrm{d}x)(\mathrm{d}x')\underline{J}_\mu(x)\big(\mathrm{Im}\,D_+^{\mu\nu}(\underline{x},\underline{x}')\big)\underline{J}_\nu(x'). \quad (16)$$

The above expression is valid for any current distribution. We consider a charged particle of charge e in the medium moving, without loss of generality, along the x^1-axis with speed v. The associated current distribution is given by

$$J^i(x) = \mathrm{e}v\delta^{i1}\delta(x^2)\delta(x^3)\delta\Big(x^1 - \frac{v}{c}x^0\Big), \quad (17)$$

$$J^0(x) = \mathrm{e}c\delta(x^2)\delta(x^3)\delta\Big(x^1 - \frac{v}{c}x^0\Big). \quad (18)$$

We work in the celebrated radiation gauge $A^0 = 0$, then the components $D_+^{ij}(x, x')$, $i, j = 1, 2, 3$, of the photon propagator satisfy, e.g., Lifshitz and Pitaevskii (1984) ($\nu = 0, 1, 2, 3,\ i, j = 1, 2, 3$) in vacuum

$$[-\partial_\nu\partial^\nu\delta^{ij} + \partial^i\partial^j]D_+^{jk}(x, x') = \delta^{ik}\delta^{(4)}(x, x'). \quad (19)$$

3. Medium of unbounded extension

For an infinite extension, the 4D delta function $\delta^{(4)}(x, x') \equiv \delta^{(4)}(x - x')$ is simply given by

$$\delta^{(4)}(x - x') = \int\frac{(\mathrm{d}Q)}{(2\pi)^4}\mathrm{e}^{\mathrm{i}Q(x-x')} \quad (20)$$

With motion along the x^1-axis, the component of the propagator of interest is $D_+^{11}(\underline{x} - \underline{x}')$ and is given by

$$D_+^{11}(\underline{x} - \underline{x}') = \int\frac{(\mathrm{d}Q)}{(2\pi)^4}\frac{\mathrm{e}^{\mathrm{i}\mathbf{Q}\cdot(\mathbf{x}-\mathbf{x}')}\mathrm{e}^{-\mathrm{i}Q^0(x^0-x'^0)/\sqrt{\mu\varepsilon}}}{\big[\mathbf{Q}^2 - Q^{0^2} - \mathrm{i}\delta\big]}, \quad \delta \to 0. \quad (21)$$

From the conservation of the current $\partial_\mu J^\mu(x) = 0 = \underline{\partial}_\mu\underline{J}^\mu(x)$ (see (12)), we obtain for $\langle N\rangle$

$$\langle N\rangle = \frac{\mu^{1/2}}{\varepsilon^{1/2}\hbar c^3}\int(\mathrm{d}x)(\mathrm{d}x')\Big(\mathbf{J}(x)\cdot\mathbf{J}(x') - \frac{1}{\mu\varepsilon}J^0(x)J^0(x')\Big)$$
$$\times\int\frac{\mathrm{d}^3\mathbf{Q}}{(2\pi)^3 2|\mathbf{Q}|}\mathrm{e}^{\mathrm{i}\mathbf{Q}\cdot(\mathbf{x}-\mathbf{x}')}\mathrm{e}^{-\mathrm{i}|\mathbf{Q}|(x^0-x'^0)/\sqrt{\mu\varepsilon}}, \quad (22)$$