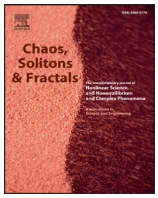
Chaos, Solitons and Fractals 88 (2016) 38–47

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos



Measuring multiscaling in financial time-series

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ARTICLE INFO

Article history:
Available online 10 December 2015

Keywords:
Multiscaling
Multifractality
Central limit theorem
Power law tails
Autocorrelation

ABSTRACT

We discuss the origin of multiscaling in financial time-series and investigate how to best quantify it. Our methodology consists in separating the different sources of measured multifractality by analyzing the multi/uni-scaling behavior of synthetic time-series with known properties. We use the results from the synthetic time-series to interpret the measure of multifractality of real log-returns time-series. The main finding is that the aggregation horizon of the returns can introduce a strong bias effect on the measure of multifractality. This effect can become especially important when returns distributions have power law tails with exponents in the range (2, 5). We discuss the right aggregation horizon to mitigate this bias.

1. Introduction

The multifractal behavior of the financial time-series has become one of the acknowledged stylized facts in the literature (see: [1–5]). Many works have been dedicated to its empirical characterization [6–16], reporting strong evidence of its presence in financial markets, and models have been proposed [17–24].

Understanding which is the origin of the measured multifractality in financial markets is still an open research challenge. This question has been raised in [25,26] where the authors pointed out that the power law tails and the autocorrelation of the analyzed time-series must be the two sources of the measured multifractality. In the first case, the multifractal behavior is a consequence of the broadness of the unconditional distribution of the returns; while in the second case, the multifractal behavior is associated with the causal structure of the time-series. It was also reported in [27] that a spurious multifractality may arise in processes with a long range autocorrelated volatility. After [25], many papers have investigated the relative contribution of these two sources to the measured multifractality [28–32], however no agreement exists. For example in [28] the author points out that the autocorrelation structure has a minor impact on the measured multifractality while the power law tails are the major source of it. In [29] they also report that the power law tails give the major contribution, but they also point out that the presence of unknown autocorrelations might introduce a negative bias effect in the quantification of multifractality. Conversely, in [30] the authors find that the autocorrelation gives the major contribution while for a specific time-series the "extreme events are actually inimical to the multifractal scaling". This lack of agreement motivated our work, leading us to investigate what the source of the measured multifractality is and how it can be detected.

In this paper we quantify the two contributions by using synthetic times-series where the two contributions can be separated. Specifically we analyze Brownian Motion with innovations drawn from a *t*-Student distribution, Multifractal Random Walk and normalized version of the Multifractal Random Walk. The measured multifractality on these synthetic series are compared with measures on both real financial log-returns and on a normalized version of the

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http://dx.doi.org/10.1016/j.chaos.2015.11.022
0960-0779/

real log-returns where the heavy tails are removed. Results show that the aggregation horizon has a strong effect on the quantification of multifractality. We verify however that there are regions of the aggregation horizon that can be used in practice to extract reliable multifractality estimators.

The rest of the paper is organized as follows: in Section 2 we perform a brief literature review introducing the tools we used for our analysis and discussing the results from previous works. In Section 3 we review the theoretical models we used and we define the multifractality estimators that shall be used throughout the paper. Sections 4 and 5 are dedicated respectively to the analysis of artificial and real financial data. In Section 6 we discuss the results and in Section 7 we conclude.

2. Background

2.1. Multifractality

Among the methods which are used for the empirical measurement of the scaling exponents, in this work we use the *Generalized Hurst Exponent method* (GHE), see [4,33,35][1] which relies on the measurement of the direct scaling of the qth-order moments of the distribution of the increments and it has been shown to be one of the most reliable estimators [34]. Let us call $X(t)$ a process with stationary increments. The GHE method considers the following function of the increments

$$E[|X(t+\tau)-X(t)|^q]=K(q)\tau^{qH(q)}, \quad (1)$$

where τ is the time horizon over which the increments are computed and $H(q)$ is the Generalized Hurst Exponent. The function $\zeta(q)=qH(q)$ is concave and $K(q)$ depends also on q. In particular, GHE considers the logarithm of Eq. (1)

$$\ln(E[|X(t+\tau)-X(t)|^q])=\zeta(q)\ln(\tau)+\ln(K(q)), \quad (2)$$

and, if linearity with respect to $\ln(\tau)$ holds, it computes the slopes of the straight lines at different q. The slopes are computed in the following way: for every q, several linear fits are computed taking $\tau \in [\tau_{min}, \tau_{max}]$, with usually $\tau_{min}=1$ and several values of τ_{max} typically between [5, 19]; the output estimator for $\zeta(q)$ is the average of these values for a given q. This method gives also the errors which are the standard deviations of these values. However, in this paper we do not perform any average over different values of τ_{min}, τ_{max} and we instead consider just one linear fit for a given range $\tau \in [\tau_{min}, \tau_{max}]$. In particular we focus our attention on two ranges, namely $\tau \in [1, 19]$, following the prescription of other works ([33,35,36]), and $\tau \in [30, 250]$. The reason for this simplification is that, given a range of τ, we did not want to weight more the small values with respect to the big values. This point will be further stressed later in the paper.

2.2. Source of multiscaling in financial data: state of the art

As already mentioned in Section 1, there is a debate in literature concerning what property of the financial time-series contributes mostly to their observed multiscaling behavior. Let us here discuss some findings present in the literature. In [28] the author studied the Dow Jones Industrial Average taken on a daily basis and processed the data in four different ways in order to uncover the source of the multiscaling behavior. The methods used were ([28]):

1. shuffling the data in order to check the impact of the shape of the unconditional distribution;
2. building up surrogate data with the same unconditional distribution and linear correlation of the empirical one but with any non linear correlation removed;
3. cutting the tails by substituting the more extreme events with resampled ones from the core of the distribution;
4. generating surrogate power law-tailed time-series, namely double Weibull and *t*-Student, preserving the temporal structure of the empirical time-series.

The author found that, on one hand the temporal structure, both linear and non linear, has a minor impact. On the other hand, the fatter the tails are, the stronger the multiscaling. And this result was confirmed both by cutting the extreme events and changing the unconditional distribution.

In [30] the authors studied again the Dow Jones Industrial Average taken on a daily basis plus the Dow Jones Euro Stoxx 50 sampled at one minute. In this case three analyses were performed:

1. shuffling the whole dataset;
2. dividing the dataset into intervals and shuffling them in order to keep short memory contributions then repeating the analysis changing the length of the intervals;
3. cutting the extreme events.

The authors found that when shuffled, the dataset loses its multiscaling behavior [30]. The shuffling of the intervals showed that the linearity of the scaling of the fluctuation functions worsen when the length of the interval is small and improves increasing it, thus according to the authors this should be regarded as a sign that .temporal correlations are the source of multiscaling. For what concerns the cut of the most extreme events they found that for the Dow Jones Industrial Average extreme events have no particular impact, while for the Dow Jones Euro Stoxx 50 they cause a distortion in the Singularity Spectrum [30].

Finally in [29] an extensive analysis was conducted on several empirical time-series including stock market indexes, exchange rates and interest rates. In order to unveil the source of the empirical multiscaling, the shuffling method was used plus a comparison with synthetic data. The authors also found an increase of the measured multiscaling of the shuffled time-series which then led them to draw two conclusions: first that the major source of the multifractality comes from the power law tails of the distribution; second that the presence of time correlations decreases the multifractality. These conclusions are consistent with the analysis of the Markov Switching Multifractal Model [19]. Further analyses have been conducted by means of fractional Brownian motions, random walks with steps drawn from a Levy distribution and ARFIMA processes, all confirming the results found on the empirical datasets ([29]).

[1] The code can be found at http://www.mathworks.com/matlabcentral/fileexchange/30076-generalized-hurst-exponent.