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Stylized facts of price gaps in limit order books

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ABSTRACT

Price gap, defined as the logarithmic price difference between the first two occupied price levels on the same side of a limit order book (LOB), is a key determinant of market depth, which is one of the dimensions of liquidity. However, the properties of price gaps have not been thoroughly studied due to the less availability of ultrahigh frequency data. In the paper, we rebuild the LOB dynamics based on the order flow data of 26 A-share stocks traded on the Shenzhen Stock Exchange in 2003. Three key empirical statistical properties of price gaps are investigated. We find that the distribution of price gaps has a power-law tail for all stocks with an average tail exponent close to 3.2. Applying modern statistical methods, we confirm that the gap time series are long-range correlated and possess multifractal nature. These three features appear to be different in the measures across stocks, but they are similar for the buy and sell LOBs within each stock. Furthermore, we also unveil buy–sell asymmetry phenomena in the properties of price gaps on the buy and sell sides of the LOBs for individual stocks. These findings deepen our understanding of the dynamics of liquidity of common stocks and can be used to calibrate agent-based computational financial models.

1. Introduction

It is a key stylized fact that asset returns over small time scales have power-law tails [1–12], which means that large price changes occur more frequently than normal distribution. This phenomenon has important application in the domain of risk management, and it is necessary to understand the origins of large price fluctuations.

It is well-documented that trading volume is a key determinant to move stock prices. The relationship between price fluctuation and trading volume over certain time period has been extensively studied [13]. There is numerous evidence showing that the magnitude of price fluctuation positively correlates to the trading volume at different time scales from one minute to one month [14–22]. The price-volume relation is usually asymmetric at the aggregate level in the sense that the price impact of a selling volume is larger than a buying volume of the same size [13]. At the transaction level, theoretical and empirical analyses show that the price impact function is nonlinear [23–37]. At the transaction level, there is no buy–sell asymmetry in the price impact function [37].

Trading volume or trade size is certainly not the solo driving force of price fluctuations. Farmer et al. found that large

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price fluctuations of stocks traded on the London Stock Exchange are essentially independent of the volume of orders, but rather driven by liquidity fluctuations characterized by the gaps between the first few occupied price levels on the opposite limit order book [38]. After investigating the TAQ data and order book data from the Island ECN, Weber and Rosenow argued that a large trading volume alone is not sufficient to explain large price changes and a lack of liquidity is a necessary prerequisite for the occurrence of large price fluctuations [39]. Naes and Skjeltorp studied the order flow data from the Oslo Stock Exchange and found that price fluctuations are positively correlated with trade number, a component of trading volume, and negatively correlated with different liquidity measures [40]. Joulin et al. analyzed the one-min data of 163 USA stocks and found that news and trading volume play a minor role in causing large price changes, and conjectured that large price fluctuations are caused by the vanishing of liquidity [41]. Based on the order flow data of Chinese stocks, Zhou found that trade size, bid–ask spread, price gaps, and outstanding volumes all play a significant role in driving price fluctuations [36].

Bid–ask spread, price gap and standing volume on the LOBs are all fundamental ingredients of liquidity. The bid–ask dynamics and price dynamics have been investigated for many financial markets [42–49]. However, only a few studies concern with the statistical properties of price gaps in financial markets. Farmer et al. analyzed the probability distribution of price gaps of a few stocks traded on London Stock Exchange and found that price gaps approximately follow a power-law distribution with the tail exponents varying from about 1.6 to 2.8 [38]. Lallouache and Abergel focused on the EUR/USD and USD/JPY foreign exchange data from the Electronic Broking Service (EBS) Spot platform. They studied the relation between the average gaps (in units of ticks) and price levels in the LOB and found that decimalized gaps decrease with the price levels in both buy and sell LOBs which do not change with time [50].

In this work, based on the order flow data of 26 A-share stocks traded on the Shenzhen Stock Exchange, we rebuild the LOBs according to the continuous double auction mechanism. We study the empirical statistical properties of the price gaps on the buy and sell LOBs. The rest of this paper is organized as follows. Section 2 briefly introduces the database we analyze. In Section 3, we investigate the probability distributions of price gaps. Section 4 estimates the memory effect of gap series using advanced statistical methods. We further investigate its multifractal nature in Section 5. Finally, we summarize the results in Section 6.

2. Dateset

Our study is based on the order flow data of 26 liquid stocks traded on the Shenzhen Stock Exchange, covering the whole year of 2003. The Shenzhen Stock Exchange adopts the continuous double auction mechanism, which was established on December 1, 1990 and started its operation on July 3, 1991. There are two kinds of independent markets on the SZSE, i.e., A-share market and B-share market. Both of them are open to mainland Chinese companies. The A-share market is traded in CNY and restricted to domestic investors, while the B-share market is traded in HKD and only open to foreign investors before February 19, 2001 since when it has been open to the domestic investors as well. Each A-share stock forms its open price through the call auction mechanism and enters the continuous double auction period since 9:30 in the morning. We focus on the data in the continuous double auction period.

There are 26 A-share stocks in our analysis, including Ping An Bank Co., Ltd. (000001), China Baoan Group Co., Ltd. (000009), CSG Holding Co., Ltd. (000012), Konka Group Co., Ltd. (000016), Shenzhen Kaifa Technology Co., Ltd. (000021), China Merchants Property Development Co., Ltd. (000024), Great Wall Computer Shenzhen Co., Ltd. (000066), Sinopec Shengli Oil Field Dynamic Group Co., Ltd. (000406), Guangdong Provincial Expressway Development Co., Ltd. (000429), Shandong Chenming Paper Holdings Co., Ltd. (000488), Guangdong Electric Power Development Co., Ltd. (000539), Foshan Electrical and Lighting Co., Ltd. (000541), Jiangling Motors Co., Ltd. (000550), Weifu High-Technology Group Co., Ltd. (000581), Chongqing Changan Automobile Co., Ltd. (000625), Hebei Iron and Steel Co., Ltd. (000709), Xinxing Ductile Iron Pipes Co., Ltd. (000778), Faw Car Co., Ltd. (000800), Shanxi Taigang Stainless Steel Co., Ltd. (000825), Citic Guoan Information Industry Co., Ltd. (000839), Wuliangye Yibin Co., Ltd. (000858), Angang Steel Co., Ltd. (000898), Hunan TV and Broadcast Intermediary Co., Ltd. (000917), Hunan Valin Steel Co., Ltd. (000932), Sinopec Zhongyuan Petroleum Co., Ltd. (000956), and Shanxi Xishan Coal and Electricity Power Co., Ltd. (000983).

The database records the order flows of the aforementioned stocks in 2003. It contains the details of order placement and order cancellation, including the order submitting time, order price, order size and order identifier which identifies whether the submitted order is a buy order, a sell order, or a cancelation. The time stamp is accurate to 0.01 s. We rebuilt the order book with respect to the placed orders and cancelled orders according to the price–time priority rule [51–53]. At each event time defined as a submitted or a cancelled order, we obtain the buy-side and sell-side LOBs, on which unexecuted limit orders occupy different price levels at $a_1, a_2, a_3, \ldots$ from low to high on the sell LOB and b_1, b_2, b_3,... from high to low on the buy LOB. The price gap $g(t)$ investigated in this work is defined as the absolute logarithmic difference between the first occupied price level (best bid or best ask) and the second occupied price level on the buy or sell LOB:

$$g(t) = \begin{cases} \ln b_1(t) - \ln b_2(t) & \text{for buy LOB} \\ \ln a_2(t) - \ln a_1(t) & \text{for sell LOB} \end{cases}. \tag{1}$$

Table 1 presents the basic statistics of the price gaps.

The second column and the ninth column of Table 1 show the order flow rate μ defined as the number of submitted order per minute. It is observed that the order flow rate varies remarkably from stock to stock, and not surprisingly μ_b strongly correlates with μ_a. We further find that $\mu_b > \mu_a$ for 4 stocks and $\mu_b < \mu_a$ for 22 stocks. The third and tenth columns present the parameter ω which is defined as the ratio of the number of gaps equaling to the tick size (0.01 CNY) to the total number of gaps on the buy or sell LOB for each stock. The value of ω varies from 0.73 to 0.99 for buy LOBs and from 0.70 to 0.99 for sell LOBs. It is not unexpected that