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A Monte Carlo multi-asset option pricing approximation for general stochastic processes

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ABSTRACT

We derived a model-free analytical approximation of the price of a multi-asset option defined over an arbitrary multivariate process, applying a semi-parametric expansion of the unknown risk-neutral density with the moments. The analytical expansion termed as the *Multivariate Generalised Edgeworth Expansion* (MGEE) is an infinite series over the derivatives of an auxiliary continuous time density. The expansion could be used to enhance a Monte Carlo pricing methodology incorporating the information about moments of the risk-neutral distribution. The efficiency of the approximation is tested over a jump-diffusion and a q-Gaussian diffusion. For the known density, we tested the multivariate lognormal (MVLN), even though arbitrary densities could be used. The MGEE relates two densities and isolates the effects of multivariate moments over the option prices. Results show that a calibrated approximation provides a good fit when the difference between the moments of the risk-neutral density and the auxiliary density are small relative to the density function of the former, and the uncalibrated approximation has immediate implications over risk management and hedging theory. The possibility to select the auxiliary density provides an advantage over classical Gram–Charlier A, B and C series approximations.

1. Introduction

The distribution of the asset returns in equity markets is ‘fat-tailed’ and ‘skewed’ [24,30]. For this reason, a semi-parametric formula like that of Jarrow and Rudd [25] profoundly impacted the literature. They approximated an arbitrary continuous risk-neutral density of a univariate asset, using a Generalised Edgeworth Expansion (GEE) over a lognormal density. To obtain the option price, they integrated the resulting approximated density under the risk-neutral measure. To calibrate the approximation, the GEE requires the empirical moments of the unknown density of the asset. By doing this, not only the price is calculated, and the moments of the asset incorporated into the final formula, but also the effects of perturbations over the moments of the distribution on the option price can be easily observed.[1]

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[1] As a result of the success of this model, it has been used in a large amount of empirical research, including Corrado and Su [13], Bhandari and Das [6] for options on portfolios, Lim et al. [35] for a parametric option pricing model, Flamouris and Giamouridis [21] for a semi-parametric model and Aït-Sahalia and Lo [1] for a non-parametric model for density estimation.

There exist popular versions of multi-asset options, one of which is the basket option: Given a vector of weights $\boldsymbol{\omega} = \{\omega_1, \ldots, \omega_n\}$, a strike price K, and a n-variate vector of assets $\mathbf{s}(t) = \{s_1, \ldots, s_n\}$, the payoff of a basket option at maturity t is $\Pi(\mathbf{s}(t), \boldsymbol{\omega}, K) = [\omega_1 s_1(t) + \cdots + \omega_n s_n(t) - K]^+$. Rainbow, quanto, spread and even index options can be regarded in the class of multi-asset options. In general, the payoff of multi-asset options can be specified as a function of the assets: $\Pi(\mathbf{s}(t), \phi, K) = [\phi(s_1(t), \ldots, s_n(t), K)]^+$, where $\phi(\,\cdot\,)$ is a multivariate real function. Special case of basket options are spread options, which are highly traded on NYMEX. The compilation made by Carmona and Durrleman [11], is a very extensive and complete reference of previous attempts and models that addressed the issue of pricing and hedging spread options. Numerical methods like Monte Carlo, binomial and trinomial trees, Fourier transform, had been used; However all methods use an approximation of the univariate density of the sum of the assets. Krekel et al. [31] did a comparison of different basket option pricing methods, concluding that Ju [26] and Beißer [5] are the best performing methods, but both methods approach the pricing through the univariate density of the distribution the payoff.

In this research, an approximate multi-asset option price is provided applying the *Multivariate Generalised Edgeworth Expansion* (MGEE) framework. In other words, we extend the results in [25] to the multivariate case. Our formula disentangles the impact of multivariate higher-order moments on the option prices.[2] It is the first time that a formula that disentangles the impact of multivariate higher-order moments on option prices has been provided.[3] The main advantage of our approximation is that it is for arbitrary processes; this means it can be used with discontinuous-time models originated not only from a Wiener diffusion, but also from Lévy processes such as jump-diffusion or q-Gaussian diffusion.[4] In the Jarrow and Rudd [25] formula the value of the European option is equal to the Black and Scholes price plus corrections based on the difference of the moments of the lognormal distribution and the real market distribution. In this paper, the GEE is extended to the multivariate case (MGEE), and then the Black and Scholes price is calculated using a Monte Carlo simulation, as there exists no equivalent closed-form Black and Scholes formula for the multivariate case. Another benefit of our results is that the moments of the risk-neutral density of the assets could be obtained separately through empirical work and, if they are available, then the price of the option is straightforwardly obtained using our formula. As a result, higher-order moment effects like the ones observed during market crashes can be easily modelled into the pricing or the hedging of the option.

The approximation provided allows us to calculate the moments of the distribution of the sum of lognormals in a multivariate setting, and this can be considered an interesting result not only for finance, but in general.[5] In [26], an univariate approximation of the risk-neutral density is provided, using a Taylor expansion over a univariate lognormal density. Kristensen and Mele [32] also provide an approximation with an application to asset pricing theory. This approximation is based on a Taylor expansion of a differential operator over the divergence between the Black and Scholes model price and the real price. Consequently, the moments of the distribution are not part of the option pricing formula, making it very difficult to understand how changes over the distribution affect the final price. Our approach for valuing multi-asset options using the multivariate risk-neutral density is novel, since all previous models attempt to price multi-asset options with a function of univariate densities: Li et al.[33,34] developed two new approximations, an original termed *second-order boundary approximation*, and an extension to the multivariate case of the Kirk [28] formula for spread options termed the *extended Kirk approximation*. Both approximations reduce the dimensions of the problem, from a multivariate integration to a function of an univariate normal standard density. In [2] the price of a spread option is approximated as the price of the sum of two compound options, and that is extended in [3] for multi-asset options. The prices of the compound options were calculated by Geske [22] and by Carr [12]. The final formula will be a function of the product of univariate densities. Working with the multivariate risk-neutral density requires additional notation from multivariate statistics. Nevertheless, the main advantage for empirical research is a more realistic framework, and it provides new tools for hedging and risk management.

The MGEE can be considered another important contribution of our research for other fields of application such as statistics. Although Perote [42] and Del Brio et al. [15] produced a *Multivariate Edgeworth Expansion* (MEE), this expansion is based on an approximation of the multivariate normal (MVN) distribution, with the complications of negative density values when the empirical density to fit is leptokurtic. We face the same risk, but if we select an appropriate distribution with skewness and kurtosis more similar to the risk-neutral density, this problem is diminished.

[2] The option price formula is derived using a Fourier inversion method. Nevertheless, the method is applied for the large class of continuous density functions with partial derivatives, resulting in a formula that is on the time domain, and there will be no need of a Fourier inversion method for pricing. In a paper by Níguez and Perote [41], a density expansion using the moments of the distribution termed General Moments Expansion (GEM) is provided. This expansion generates only positive densities; however, it needs an additional vector of parameters of the same dimension of the distribution dimension; these additional parameters have no economical significance.

[3] Schlögl [44] provides an multi-asset option approximation using a multivariate Gram–Charlier A series expansion; however, there are assumptions over the risk-neutral density, and an additional methodology is needed to extract the moments inside the expansion from the Hermite polynomials.

[4] Our results complement the results of Filipović et al. [20], as we provide a thorough study of the higher-order moments effect over option prices. In [29] a Gram–Charlier expansion is derived for pricing options using the first four moments of a univariate risk-neutral distribution.

[5] Limpert et al. [36] and Dufresne [18] review the importance of the distribution of the sum of lognormals in finance, and in physical sciences in general.