Chaos, Solitons and Fractals 86 (2016) 64–74

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

On the propagation of solitons in ferrites: The inverse scattering approach

Hermann T. Tchokouansi[a,b,c,*], Victor K. Kuetche[a,b,c,d], Timoleon C. Kofane[b,c,d]

[a] National Advanced School of Engineering, University of Yaounde I, P.O. Box 8390, Cameroon
[b] Department of Physics, Faculty of Science, University of Yaounde I, P.O. Box 812, Cameroon
[c] Centre d'Excellence en Technologies de l'Information et de la Communication (CETIC), University of Yaounde I, P.O. Box 812, Yaounde, Cameroon
[d] The Abdus Salam International Centre for Theoretical Physics (ICTP), Strada Costiera, Trieste 11-I-34151, Italy

A R T I C L E I N F O

Article history:
Received 27 August 2015
Revised 16 February 2016
Accepted 23 February 2016
Available online 12 March 2016

PACS:
05.45.Yv

Keywords:
Kraenkel–Manna–Merle system
Lax-pairs
Inverse scattering transform
Soliton solutions

A B S T R A C T

We pay particular attention to a nonlinear evolution system derived by Kraenkel et al. (2000) from Maxwell's equations and the Landau–Lifshitz–Gilbert equation, describing the propagation of ultra-short wave in ferromagnetic materials. Since the associated Lax-pairs of such a system has been provided standing for a proof of its integrability, we follow the inverse scattering transform method and particularly the Wadati–Konno–Ichikawa scheme to unveil the soliton solutions to this system and study their scattering properties.

1. Introduction

The investigation of the integrability of nonlinear evolution systems has attracted many attentions over previous years. As a consequence, number of researchers have developed various approaches to construct solutions to such systems, namely, Hirota's bilinearization [1], the Darboux transformation [2], the inverse scattering transform [3], just to name a few. Between these methods, Hirota's bilinearization is a straightforward one, that uses associated equations along with differential operators known as Hirota's operators linked with some transformations. The success of this method leads on the facility in generating multisoliton solutions. But it is not always easy to find appropriate transformations necessary to the generation of the bilinear equation associated to an evolution system. This situation suggests to look for alternative methods to solve nonlinear evolution systems, namely, the inverse scattering method. Paying particular interest to this method, number of evolution equations have been investigated using this tool. This technique, to be performed, one needs foremost to determine associated Lax-pairs to the nonlinear evolution system, useful to construct its soliton solutions. Therefore, it is necessary to use another tool to find these associated Lax-pairs. Such a task can be done directly, or by using an efficient method often undertaken to investigate the integrability of nonlinear evolution systems, known in the literature as the prolongation structure analysis due to Wahlquist and Estabrook [4]. This method, based on the language of differential geometry and Lie-algebra representation theory, is a straightforward method to find Lax-pairs of nonlinear wave equations and in addition, provides the hidden structural symmetries embedded in such systems. Once the Lax-formulation of a given nonlinear equation is settled, the inverse scattering method can be performed. There are various approaches that can be handled, depending on the form of the Lax-pairs. The most known are the Ablowitz–Kaup–Newell–Segur and the Wadati–Konno–Ichikawa (WKI)-schemes [5–7], just to name a few.

Concerning the problem of the integrability of nonlinear evolution equations, it sometimes appears that, from the viewpoint of some methods, these systems are integrable and are not in the viewpoint of other methods, making it necessary to investigate the solution using different methods. Therefore, since the system derived by Kraenkel et al. [8] has been investigated using Hirota's bilinear method, one- and two-soliton solutions being provided, the question that arises is the one to know whether this system is also solvable using the inverse scattering method. Thus, we organize this paper as follows: in Section 2, we investigate the eigenvalue

* Corresponding author at: National Advanced School of Engineering, University of Yaounde I, P.O. Box 8390, Cameroon. Tel.: +237 679214381/695655562.

E-mail addresses: tchokouansi@yahoo.fr, tchokouansihermann@gmail.com (H.T. Tchokouansi), vkuetche@yahoo.fr (V.K. Kuetche), tckofane@yahoo.com (T.C. Kofane).

http://dx.doi.org/10.1016/j.chaos.2016.02.032

problem of the system derived by Kraenkel et al. [8] in details. Next in Section 3, we construct one-, two- and three-soliton solutions to this system. In Section 4, we discuss the results obtained while addressing some physical implications. Finally, we end this work with a brief summary and perspectives.

2. Inverse scattering transform of the Kraenkel–Manna–Merle (KMM) system

While investigating the propagation of short-waves in saturated ferromagnetic materials with zero-conductivity in the presence of an external field, Kraenkel et al. [8] have constructed the following nonlinear evolution system:

$$B_{xt} = BC_x - sB_x, \tag{1a}$$

$$C_{xt} = -BB_x \tag{1b}$$

from Maxwell's equations supplemented by the Landau–Lifshitz–Gilbert equation [9,10], where the quantities B and C represent two physical observables standing for the magnetization and the external magnetic field related to the ferrite, respectively. The parameter s stands for the damping effect, while the subscripts t and x are denoting partial derivatives according to the time-like and space-like variables respectively. Considering the zero damping effect ($s = 0$), Kraenkel et al. [8] provided via some transformations, one-soliton solution to the above system. Going forward, Nguepjouo et al. [10] and separately Kuetche et al. [11,12], after having investigated the phase portrait analysis of system (1), have investigated its integrability properties under the framework of prolongation structure and have provided associated Lax-pairs, opening the way in looking for it soliton solutions via the inverse scattering method. But, instead of using the previously mentioned method, they derived the related Hirota's bilinearization to system (1), and constructed its one- and two-soliton solutions while studying in details their scattering properties. Therefore, the study of the Lax-pairs derived by Kuetche et al. [11,12] is worth underlying.

Investigating the prolongation structure of system (1), Kuetche et al. [11] showed that the system is integrable under the condition ($s = 0$). The associated Lax-pairs have therefore been provided, having as expression

$$y_x = \imath\lambda \begin{pmatrix} C_x & B_x \\ B_x & C_x \end{pmatrix} y, \tag{2a}$$

$$y_t = \begin{pmatrix} 1/4\imath\lambda & -B/2 \\ B/2 & -1/4\imath\lambda \end{pmatrix} y, \tag{2b}$$

which give rise to Eq. (1) under the cross differentiation $y_{xt} = y_{tx}$, implying that the spectral parameter λ is a constant. System (2) then constitutes the starting point to the investigation of the inverse scattering transform method. According to the expression of the above Lax-pairs, we suitably follow the WKI-scheme [13].

Indeed, we define the associated Jost functions as follows:

$$\left.\begin{aligned} \phi &\to \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(-\imath\lambda x), \\ \bar{\phi} &\to \begin{pmatrix} 0 \\ -1 \end{pmatrix} \exp(\imath\lambda x), \end{aligned}\right\} \text{as} \quad x \to -\infty, \tag{3}$$

and

$$\left.\begin{aligned} \psi &\to \begin{pmatrix} 0 \\ 1 \end{pmatrix} \exp(\imath\lambda x), \\ \bar{\psi} &\to \begin{pmatrix} 1 \\ 0 \end{pmatrix} \exp(-\imath\lambda x) \end{aligned}\right\} \text{as} \quad x \to +\infty. \tag{4}$$

The functions $\phi, \bar{\phi}, \psi$ and $\bar{\psi}$ being related by scattering coefficients as follows:

$$\phi = a\bar{\psi} + b\psi, \tag{5a}$$

$$\bar{\phi} = -\bar{a}\psi + \bar{b}\bar{\psi}, \tag{5b}$$

where $a, \bar{a}, b, \bar{b}$ verify

$$a\bar{a} + b\bar{b} = 1. \tag{6}$$

It is important to mention that for the complex-valued λ we have:

$$\begin{pmatrix} \bar{\phi}_1(\lambda) \\ \bar{\phi}_2(\lambda) \end{pmatrix} = \begin{pmatrix} \phi_2^\star(\lambda^\star) \\ -\phi_1^\star(\lambda^\star) \end{pmatrix}, \tag{7a}$$

$$\begin{pmatrix} \bar{\psi}_1(\lambda) \\ \bar{\psi}_2(\lambda) \end{pmatrix} = \begin{pmatrix} \psi_2^\star(\lambda^\star) \\ -\psi_1^\star(\lambda^\star) \end{pmatrix}, \tag{7b}$$

from which one deduces naturally the relations

$$\bar{a}(\lambda) = a^\star(\lambda^\star), \qquad \bar{b}(\lambda) = b^\star(\lambda^\star). \tag{8}$$

We now investigate the asymptotic behavior of the Jost functions for large λ under the following boundary conditions:

$$\left.\begin{aligned} C_x &\to 1 \\ B_x &\to 0 \end{aligned}\right\} \quad as \quad |x| \to \infty. \tag{9}$$

Defining the quantity

$$\phi_1 = \exp\left(-\imath\lambda x + \int_{-\infty}^{x} \sigma(\lambda, l)\mathrm{d}l\right), \tag{10}$$

and substituting Eq. (10) into (2), we obtain

$$\sigma_t = \frac{1}{2}\left[\left(C_x + 1 - \frac{\sigma}{\imath\lambda}\right)\frac{B}{B_x}\right]_x, \tag{11a}$$

$$\begin{aligned} &\sigma_x + \sigma^2 - 2\imath\lambda\sigma - \lambda^2 \\ &= \frac{B_{xx}}{B_x}\sigma + \imath\lambda\frac{B_{xx}(B_x - 1)}{B_x} - \lambda^2(C_x^2 + B_x^2), \end{aligned} \tag{11b}$$

where it appears clearly that σ is a conserved quantity. Let us now expand σ in power series of $\imath\lambda$ as follows:

$$\sigma = \sum_{j=-1}^{\infty} \sigma_j(\imath\lambda)^{-j}, \tag{12}$$

which substituted in Eq. (12) yields

$$\begin{aligned} &\sigma_{n,x} + \sum_{j=-1}^{n+1} \sigma_j\sigma_{n-j} - 2\sigma_{n+1} - \left(\frac{B_{xx}}{B_x}\right)\sigma_n - \frac{B_{xx}(B_x - 1)}{B_x}\delta_{n,-1} \\ &= (C_x^2 + B_x^2 - 1)\delta_{n,-2}. \end{aligned} \tag{13}$$

Infinite number of conserved quantities can hence be generated, the first two of them being given as follows:

$$\sigma_{-1} = 1 - \epsilon\sqrt{C_x^2 + B_x^2}, \tag{14a}$$

$$\sigma_0 = \frac{1}{2(1 - \sigma_{-1})}\left[\sigma_{-1,x} - B_x\left(\frac{C_x}{B_x}\right)_x\right], \tag{14b}$$

with $\epsilon = \pm 1$. We then provide the asymptotic behavior of the function ϕ as

$$\begin{aligned} \phi &= \begin{pmatrix} 1 \\ -\frac{C_x + \sqrt{C_x^2 + B_x^2}}{B_x} \end{pmatrix} \\ &\quad \times \exp\left[-\imath\lambda x + \imath\lambda\int_{-\infty}^{x} \sigma_{-1}\mathrm{d}l + \int_{-\infty}^{x} \sigma_0\mathrm{d}l\right] + o(1/\lambda), \end{aligned} \tag{15}$$