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Stochastic chaos in a Duffing oscillator and its control

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Abstract

Stochastic chaos discussed here means a kind of chaotic responses in a Duffing oscillator with bounded random parameters under harmonic excitations. A system with random parameters is usually called a stochastic system. The modifier 'stochastic' here implies dependent on some random parameter. As the system itself is stochastic, so is the response, even under harmonic excitations alone. In this paper stochastic chaos and its control are verified by the top Lyapunov exponent of the system. A non-feedback control strategy is adopted here by adding an adjustable noisy phase to the harmonic excitation, so that the control can be realized by adjusting the noise level. It is found that by this control strategy stochastic chaos can be tamed down to the small neighborhood of a periodic trajectory or an equilibrium state. In the analysis the stochastic Duffing oscillator is first transformed into an equivalent deterministic nonlinear system by the Gegenbauer polynomial approximation, so that the problem of controlling stochastic chaos can be reduced into the problem of controlling deterministic chaos in the equivalent system. Then the top Lyapunov exponent of the equivalent system is obtained by Wolf's method to examine the chaotic behavior of the response. Numerical simulations show that the random phase control strategy is an effective way to control stochastic chaos.

1. Introduction

Chaos, widely observed in nonlinear dynamic systems [1], is one of the most important scientific discoveries in the last century. The basic characteristic of chaos is the repellency of any two adjacent trajectories of chaos, which results in a positive top Lyapunov exponent of the system. It is well known that embedded in the vicinity of a chaotic attractor are numerous unstable periodic orbits, most of which correspond to saddles in the Poincare maps, forming potential crises to chaos. In any occasion one of the unstable periodic orbits changing its stability will bring disaster to chaos. Feedback control is an effective means to change system characteristics. It is Ott et al. [2–4], who successfully tamed chaos by feedback control to make it settle down onto a desired stabilized periodic orbit, formally unstably embedded in the chaotic manifold. Another way to quench chaos may be drawn experience of asynchronous

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quenching of self-excited oscillations in a van der Pol oscillator, which is an open-loop control, simply by adding some exotic disturbance [5–9] to the system. Among those non-feedback control strategies to suppress chaos, the random phase control is an interesting one [9]. The control of stochastic chaos we discuss in this paper is mainly based on this kind of chaos control.

As to stochastic chaos, we mean chaotic responses in a nonlinear system with random parameters under harmonic excitations. Since the system per se depends upon some random parameters, so does its response. So far there is no generally acknowledged definition for deterministic chaos, let alone for stochastic chaos. However, there is a common understanding that possessing at least one positive Lyapunov exponent is a necessary condition (and usually also a sufficient one in most cases) for deterministic chaos. On the other hand, stochastic chaos is actually an ensemble of infinite numbers of deterministic chaos. Namely, every sample motion of stochastic chaos is deterministic chaos. Suppose that the random parameters can be modelled by bounded random variables with so-called λ-PDF, then the response problem of a stochastic system can be solved in the following way [10]. Under the assumption that the random parameters are statistically independent of the excitation, then the expression for response of a stochastic system can be separated in time domain and random space domain, namely in a series of products of a time function and an orthogonal function of random parameters. By the associated Gegenbauer polynomial approximation the stochastic system can be reduced into an equivalent deterministic system, which determines the time functions of the response alone. If the response time functions of an equivalent deterministic system are found to be deterministic chaos, then the response of the original stochastic system should be stochastic chaos. Thus, the control of stochastic chaos in a stochastic system can be reduced into the control of deterministic chaos in its equivalent system.

In this paper, we are going to explore the control of stochastic chaos in a Duffing oscillator with bounded random parameters. The paper is organized as follows. As preliminary knowledge to deal with the response problem of a stochastic system with bounded random parameters, λ-PDF and Gegenbauer polynomials are introduced in Section 2. Transformation of the stochastic Duffing system into its equivalent deterministic system by Gegenbauer polynomial approximation is illustrated in Section 3. Followed is Section 4, devoted to exploring stochastic chaos in the stochastic Duffing system and realizing its control by a random phase control strategy. Finally, conclusions are drawn with discussions.

2. λ-PDF and Gegenbauer polynomials

A family of bounded probability density functions with mono-peak and symmetrically distributed within $[-1,1]$, named as λ-PDF can be defined as follows [10]:

$$p_\lambda(\xi) = \begin{cases} \rho_\lambda(1-\xi^2)^{\lambda-\frac{1}{2}} & |\xi| \leqslant 1 \\ 0 & |\xi| > 1 \end{cases} \tag{1}$$

where $\lambda \geqslant 0$ is a parameter, and the normalizing coefficient ρ_λ can be written as

$$\rho_\lambda = \frac{\Gamma(\lambda+1)}{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\lambda+\frac{1}{2}\right)} \tag{2}$$

in which $\Gamma(\lambda)$ is a Gamma function. Any probability density function with mono-peak and symmetrically distributed within $[-1,1]$ can be approximated by λ-PDF, provided that it is continuous and smooth. The graph of λ-PDF for different values of λ is shown in Fig. 1.

As a family of orthogonal functions, Gegenbauer polynomials can be expressed as [11,12]

$$G_n^\lambda(\xi) = \sum_{k=0}^{n} a_{n,k}^\lambda \left(\frac{\xi-1}{2}\right)^k, \quad n = 0, 1, 2, \ldots \tag{3}$$

where $a_{n,k}^\lambda$ can be written as

$$a_{n,k}^\lambda = \frac{1}{k!(n-k)!}\frac{(2\lambda)_n(2\lambda+n)_k}{(\lambda+1/2)_k} \tag{4}$$

in which $(\lambda)_k$ is the Gaussian symbol, which stands for

$$(\lambda)_k = \lambda(\lambda+1)\cdots(\lambda+k-1)$$