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# Chaos control of chaotic dynamical systems using backstepping design

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**Abstract**

This work presents chaos control of chaotic dynamical systems by using backstepping design method. This technique is applied to achieve chaos control for each of the dynamical systems Lorenz, Chen and Lü systems. Based on Lyapunov stability theory, control laws are derived. We used the same technique to enable stabilization of chaotic motion to a steady state as well as tracking of any desired trajectory to be achieved in a systematic way. Numerical simulations are shown to verify the results.


## 1. Introduction

Dynamic chaos is a very interesting nonlinear effect which has been intensively studied during the last two decades. The effect is very common, it has been detected in a large number of dynamic systems of various physical nature. However, this effect is usually undesirable in practice, and it restricts the operating range of many electronic and mechanic devices. Recently, controlling this kind of complex dynamical systems has attracted a great deal of attention within the engineering society. Chaos control, in a broader sense, can be divided into two categories [1]: one is to suppress the chaotic dynamical behavior and the other is to generated or enhance chaos in nonlinear systems (known as chaotincation or anti-control of chaos [2,3]). Nowadays, different techniques and methods have been proposed to achieve chaos control. For instance, OGY method [4], differential geometric method [5], feedback and nonfeedback control [6–9], inverse optimal control [10], adaptive control [11,12] and backstepping design technique [13]. In 1963, Lorenz [14] found the first canonical chaotic attractor, which has just been mathematically confirmed to exist [15]. In 1999, Chen [2] found another similar but topologically not equivalent chaotic attractor, as the dual of the Lorenz system, in a sense defined by Vanêĉek and Ĉelikovskŷ [16]: the Lorenz system satisfies the condition $a_{12}a_{21} > 0$ while Chen system satisfies $a_{12}a_{21} < 0$, where $a_{12}$, $a_{21}$ are the corresponding elements in the constant matrix $A = (a_{ij})_{3\times 3}$ for the linear part of the system. Very recently, Lü and Chen [17–19] found a new chaotic system, bearing the name of the Lü system, which satisfies the condition $a_{12}a_{21} = 0$, thereby bridging the gap between the Lorenz and Chen attractors [18,19].

In this work, chaos in Lorenz, Chen and Lü systems is controlled by using backstepping design method. At the same time we used the same method to enable stabilization of chaotic motion to a steady state as well as tracking of any

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desired trajectory to be achieved in a systematic way. Computer simulation is also given for the purpose of illustration and verification.

## 2. Controlling Lorenz system

The Lorenz system is described by

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = cx - xz - y, \\ \dot{z} = xy - bz, \end{cases} \tag{1}$$

which has a chaotic attractor when $a = 10$, $b = 8/3$, $c = 28$. We will use backstepping method to design a controller. In order to control Lorenz system we add a control input $u_1$ to the third equation of system (1). Then the controlled Lorenz system is

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = cx - xz - y, \\ \dot{z} = xy - bz + u_1. \end{cases} \tag{2}$$

Our objective is to find a control law $u_1$ for stabilizing the state of the controlled system (2) at a bounded point.

Starting from the first equation, a stabilizing function $\alpha_1(x)$ has to be designed for the virtual control $y$ in order to make the derivative of $V_1(x) = \frac{x^2}{2}$, i.e., $\dot{V}_1(x) = -ax_2 + axy$ be negative definite. Assume that $\alpha_1(x) = px$ and define an error variable

$$\bar{y} = y - \alpha_1(x) \tag{3}$$

Then we obtained the $(x, \bar{y})$-subsystem

$$\begin{cases} \dot{x} = a\bar{y} - a(1 - p)x, \\ \dot{\bar{y}} = cx - xz - \bar{y} - px - ap\bar{y} + ap(1 - p)x. \end{cases} \tag{4}$$

We can construct a Lyapunov function as follows:

$$V_2(x, \bar{y}) = V_1(x) + \frac{1}{2}\bar{y}^2.$$

Calculating the time derivative of $V_2(x, \bar{y})$ along system (4), we have

$$\dot{V}_2 = -a(1 - p)x^2 - (1 + ap)\bar{y}^2 - x\bar{y}[z - a - c + p - ap(1 - p)].$$

We can choose

$$z = \alpha_2(x, \bar{y}) = a + c - p + ap(1 - p).$$

Apparently, $\dot{V}_2$ is negative definite if $\frac{-1}{a} < p < 1$. Similarly, let

$$\bar{z} = z - \alpha_2(x, \bar{y}), \tag{5}$$

then we get the following system in the $(x, \bar{y}, \bar{z})$ coordinates

$$\begin{cases} \dot{x} = a\bar{y} - a(1 - p)x, \\ \dot{\bar{y}} = cx - xz - \bar{y} - px - ap\bar{y} + ap(1 - p)x, \\ \dot{\bar{z}} = x\bar{y} + px^2 - b\bar{z} - b[a + c - p + ap(1 - p)] + u_1. \end{cases} \tag{6}$$

We can construct a Lyapunov function as follows:

$$V_3(x, \bar{y}, \bar{z}) = V_2(x, \bar{y}) + \frac{1}{2}\bar{z}^2.$$

Calculating the time derivative of $V_3(x, \bar{y}, \bar{z})$ along system (6), we have

$$\dot{V}_3 = -a(1 - p)x^2 - (1 + ap)\bar{y}^2 - b\bar{z}^2 + \bar{z}[px^2 - b(a + c - p + ap(1 - p)) + u_1]$$