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Chaos synchronization between two different chaotic dynamical systems

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Abstract

This work presents chaos synchronization between two different chaotic systems by nonlinear control laws. First, synchronization problem between Genesio system and Rossler system has been investigated, and then the similar approach is applied to the synchronization problem between Genesio system and a new chaotic system developed recently in the literature. The control performances are verified by two numerical examples.

1. Introduction

During the last two decades, synchronization in chaotic dynamic systems has received a great deal of interest among scientists from various research fields [2–24] since Pecora and Carroll [1] introduced a method to synchronization two identical chaotic systems with different initial conditions. The idea of synchronization is to use the output of the master system to control the slave system so that the output of the response system follows the output of the master system asymptotically. A wide variety of approaches have been proposed for the synchronization of various chaotic systems which include PC method [1], OGY method [3,20], active control approach [12], adaptive control method [15,17–19], time-delay feedback approach [4], and backstepping design technique [9], etc. However, most of the methods mentioned above synchronize two identical chaotic systems. In fact, in many practical world such as laser array and biological systems, it is hardly the case that every component can be assumed to be identical. As a result, more and more applications of chaos synchronization in secure communications make it much more important to synchronize two different chaotic systems in recent years [23]. In this regard, some works on synchronization of two different chaotic systems have been performed [22,23].

This article considers the synchronization problems of two different chaotic systems. The Genesio system [6,21] is taken as drive system. The Rossler system and a new chaotic system devised by Chen and Lee [24] are selected as response system. In general, it is known that nonlinear control is an effective method for making two different chaotic systems be synchronized. Thus, a class of nonlinear control scheme is applied to solve the synchronization problems

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of this work. Then, the stability of error signals between drive and response system is easily proved by linear control theory.

This article is organized as follows: In Section 2, the synchronization problem between Genesio system and Rossler system is considered. The result in Section 2 is extended to the problem between Genesio system and a new chaotic system in Section 3. In Section 4, we provide two numerical examples to demonstrate the effectiveness of the proposed method. Finally concluding remark is given in Section 5.

2. Synchronization between Genesio and Rossler systems

In order to observe the chaos synchronization behavior in Genesio and Rossler systems, it is assumed that Genesio system drives the Rossler system. Thus, the drive and response systems are as follows:

$$\begin{cases} \dot{x} = y, \\ \dot{y} = z, \\ \dot{z} = -ax - by - cz + x^2, \end{cases} \tag{1}$$

and

$$\begin{cases} \dot{x}_1 = -y_1 - z_1 + u_1, \\ \dot{y}_1 = x_1 + b_1 y_1 + u_2, \\ \dot{z}_1 = a_1 + z_1(x_1 - c_1) + u_3, \end{cases} \tag{2}$$

where a, b, c, a_1, b_1 and c_1 are positive scalars, and u_1, u_2 and u_3 are the control inputs to be designed. The aim of this section is to determine the control laws u_i for the global synchronization of two different chaotic systems. The Genesio system is chaotic for the parameters, $a = 6$, $b = 2.92$, $c = 1.2$, and the Rossler system is chaotic for the parameters, $a_1 = 0.2$, $b_1 = 0.2$, $c_1 = 5.7$.

For chaotic synchronization of above two different systems, the error dynamical system is described by

$$\begin{cases} \dot{e}_1 = -y_1 - z_1 - y + u_1, \\ \dot{e}_2 = x_1 + b_1 y_1 - z + u_2, \\ \dot{e}_3 = a_1 + z_1(x_1 - c_1) + ax + by + cz - x^2 + u_3, \end{cases} \tag{3}$$

where

$$\begin{cases} e_1(t) = x_1(t) - x(t), \\ e_2(t) = y_1(t) - y(t), \\ e_3(t) = z_1(t) - z(t). \end{cases} \tag{4}$$

Now we define the control functions u_1,u_2 and u_3 as follows:

$$\begin{cases} u_1 = 2y_1 + z_1 - e_1, \\ u_2 = z - x_1 - 2b_1 y_1 + b_1 y, \\ u_3 = -a_1 + x^2 - z_1 x_1 - ax - by - cz_1 + c_1 z_1. \end{cases}$$

Hence the error system (3) becomes

$$\begin{cases} \dot{e}_1 = -e_1 + e_2, \\ \dot{e}_2 = -b_1 e_2, \\ \dot{e}_3 = -ce_3. \end{cases} \tag{5}$$

The error system (5) is a linear system of the form, $\dot{w} = Aw$. Thus by linear control theory, if the system matrix A is Hurwitz, the system is asymptotically stable. Hence the error system (5) with

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -b_1 & 0 \\ 0 & 0 & -c \end{bmatrix} \tag{6}$$