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Propagating wave patterns and "peakons" of the Davey–Stewartson system

K.W. Chow [a,*], S.Y. Lou [b]

[a] Department of Mechanical Engineering, University of Hong Kong, Pokfulam, Hong Kong
[b] Department of Physics, Shanghai Jiao Tong University, Shanghai 200030, People's Republic of China

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Abstract

Two exact, doubly periodic, propagating wave patterns of the Davey–Stewartson system are computed analytically by a special separation of variables procedure. For the first solution there is a cluster of smaller peaks within each period. The second one consists of a rectangular array of 'plates' joined together by sharp edges, and is thus a kind of 'peakons' for this system of (2 + 1) (2 spatial and 1 temporal) dimensional evolution equations. A long wave limit will yield exponentially localized waves different from the conventional dromion. The stability properties and nonlinear dynamics must await further investigations.

1. Introduction

The Davey–Stewartson model (DS) is an important system of evolution equations, both from the perspectives of theory and applications. DS can arise in hydrodynamics [1] and plasma physics [2]. Theoretically, many techniques of the modern theory of nonlinear waves are relevant, e.g., special Hirota bilinear forms [3], Darboux transformations [4], symmetries [5], rich soliton and related structures [6,7]. We shall take DS as

$$\begin{aligned} &\mathrm{i}\frac{\partial A}{\partial t}+\frac{1}{2}\left(\frac{\partial^2 A}{\partial \xi^2}+\frac{\partial^2 A}{\partial \eta^2}\right)+vA^2A^*=QA,\\ &\frac{\partial^2 Q}{\partial \xi^2}-\frac{\partial^2 Q}{\partial \eta^2}=2v\frac{\partial^2}{\partial \xi^2}(AA^*). \end{aligned} \tag{1.1}$$

In the hydrodynamic context, A is the envelope of the wave packet while Q is the induced mean flow. New coordinates X, Y are defined by

$$\xi=\frac{X+Y}{\sqrt{2}},\quad \eta=\frac{X-Y}{\sqrt{2}} \tag{1.2}$$

* Corresponding author. Tel.: +852 2859 2641; fax: +852 2858 5415.
E-mail addresses: kwchow@hkusua.hku.hk (K.W. Chow), sylou@sjtu.edu.cn (S.Y. Lou).

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and the transformed DS considered in the present work will be

$$\begin{aligned} &\mathrm{i}\frac{\partial A}{\partial t}+\frac{1}{2}\left(\frac{\partial^2 A}{\partial X^2}+\frac{\partial^2 A}{\partial Y^2}\right)+vA^2A^*=QA, \\ &2\frac{\partial^2 Q}{\partial X\partial Y}=v\left(\frac{\partial}{\partial X}+\frac{\partial}{\partial Y}\right)^2(AA^*). \end{aligned} \tag{1.3}$$

Recently a class of novel, exact solutions of DS and other (2 + 1) (2 spatial and 1 temporal) dimensional nonlinear evolution equations can be obtained by a special separation of variables approach [8]. More precisely, one exact solution of the system (1.3) is

$$\begin{aligned} &p=p(x),\quad q=q(y),\quad p_0=p_0(x),\quad q_0=q_0(y),\quad r=r(x),\quad s=s(y), \\ &Q=p_0+q_0-\frac{p_{xx}+q_{yy}}{a_0+p+q}+\frac{(p_x+q_y)^2}{(a_0+p+q)^2}, \\ &A=\sqrt{\frac{2}{v}}\left(\frac{\sqrt{p_xq_y}}{a_0+p+q}\right)\exp(\mathrm{i}(r+s)),\quad r_x=c_1+\frac{\zeta}{p_x},\quad s_y=c_2-\frac{\zeta}{q_y}, \\ &p_0=\frac{p_{xxx}}{4p_x}-\frac{p_{xx}^2}{8p_x^2}-\frac{\delta}{8}+\frac{c_1^2}{2}-\frac{\zeta^2}{2p_x^2},\quad q_0=\frac{q_{yyy}}{4q_y}-\frac{q_{yy}^2}{8q_y^2}+\frac{\delta}{8}+\frac{c_2^2}{2}-\frac{\zeta^2}{2q_y^2}. \end{aligned} \tag{1.4}$$

$$x=X-c_1t,\quad y=Y-c_2t \tag{1.5}$$

are the coordinates translating with the wave pattern. a_0, c_1, c_2, ζ are constants, and the relevant functions depend on the indicated variables only. More complicated solutions with terms involving products of p and q in the denominator can be constructed, but details will be left for future studies.

The choice of exponential functions as the basis functions in (1.4) leads to generalized solutions of localized solitons or dromions. The purpose of the present note is to demonstrate that the choice of the Jacobi elliptic functions as building blocks (or $p'(x)$, $q'(y)$ in (1.4)) is feasible too, and will result in doubly periodic, propagating wave patterns for DS. Two constraints will dictate the choice of elliptic functions. Firstly, the building block functions need to be nonnegative as a square root is taken in the process. Simple choices like the functions sn and cn, which oscillate in a sinusoidal manner, must be rejected. Secondly, for analytical convenience, we restrict the attention to simple cases where both A and Q can be evaluated in simple, closed forms which do not involve the elliptic integrals in this paper. The selections of the Jacobi elliptic function dn [9,10] and its reciprocal will satisfy these requirements, and will now lead to these two new wave patterns for DS (Section 2). Further properties like the long wave limit and the boundary conditions will also be investigated (Section 3).

2. Doubly periodic wave patterns

2.1. First solution

By choosing both basis functions $p'(x)$, $q'(y)$ in (1.4) in the x, y directions as 'dn', we obtain

$$A=\sqrt{\frac{2}{v}}\frac{\sqrt{\mathrm{dn}(\alpha x,k)\mathrm{dn}(\beta y,k_1)}}{R_1}\exp(\mathrm{i}U_1), \tag{2.1}$$

$$U_1=c_1x-\frac{\zeta}{\alpha\sqrt{1-k^2}}\sin^{-1}\left(\frac{\mathrm{cn}(\alpha x,k)}{\mathrm{dn}(\alpha x,k)}\right)+c_2y+\frac{\zeta}{\beta\sqrt{1-k_1^2}}\sin^{-1}\left(\frac{\mathrm{cn}(\beta y,k_1)}{\mathrm{dn}(\beta y,k_1)}\right), \tag{2.2}$$

$$R_1=a_0+\frac{1}{\alpha}\sin^{-1}[\mathrm{sn}(\alpha x,k)]+\frac{1}{\beta}\sin^{-1}[\mathrm{sn}(\beta y,k_1)], \tag{2.3}$$

$$\begin{aligned} Q=&\frac{k^2\alpha^2}{4}\left(\mathrm{sn}^2(\alpha x,k)-\mathrm{cn}^2(\alpha x,k)-\frac{k^2\mathrm{sn}^2(\alpha x,k)\mathrm{cn}^2(\alpha x,k)}{2\mathrm{dn}^2(\alpha x,k)}\right)+\frac{c_1^2}{2}-\frac{\zeta^2}{2\mathrm{dn}^2(\alpha x,k)} \\ &+\frac{k_1^2\beta^2}{4}\left(\mathrm{sn}^2(\beta y,k_1)-\mathrm{cn}^2(\beta y,k_1)-\frac{k_1^2\mathrm{sn}^2(\beta y,k_1)\mathrm{cn}^2(\beta y,k_1)}{2\mathrm{dn}^2(\beta y,k_1)}\right)+\frac{c_2^2}{2}-\frac{\zeta^2}{2\mathrm{dn}^2(\beta y,k_1)} \\ &+\frac{[k^2\alpha\mathrm{sn}(\alpha x,k)\mathrm{cn}(\alpha x,k)+k_1^2\beta\mathrm{sn}(\beta y,k_1)\mathrm{cn}(\beta y,k_1)]}{R_1}+\frac{[\mathrm{dn}(\alpha x,k)+\mathrm{dn}(\beta y,k_1)]^2}{R_1^2}. \end{aligned} \tag{2.4}$$