Chaos, Solitons and Fractals 85 (2016) 57–67

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Pattern formation and control of spatiotemporal chaos in a reaction diffusion prey–predator system supplying additional food

Santu Ghorai, Swarup Poria*

Department of Applied Mathematics, University of Calcutta, 92 APC Road, Kolkata 700009, India

A R T I C L E I N F O

Article history:
Received 29 September 2015
Revised 22 December 2015
Accepted 5 January 2016
Available online 11 February 2016

Keywords:
Spatiotemporal pattern
Spatiotemporal chaos
Hopf–Turing region
Additional food

A B S T R A C T

Spatiotemporal dynamics of a predator–prey system in presence of spatial diffusion is investigated in presence of additional food exists for predators. Conditions for stability of Hopf as well as Turing patterns in a spatial domain are determined by making use of the linear stability analysis. Impact of additional food is clear from these conditions. Numerical simulation results are presented in order to validate the analytical findings. Finally numerical simulations are carried out around the steady state under zero flux boundary conditions. With the help of numerical simulations, the different types of spatial patterns (including stationary spatial pattern, oscillatory pattern, and spatiotemporal chaos) are identified in this diffusive predator–prey system in presence of additional food, depending on the quantity, quality of the additional food and the spatial domain and other parameters of the model. The key observation is that spatiotemporal chaos can be controlled supplying suitable additional food to predator. These investigations may be useful to understand complex spatiotemporal dynamics of population dynamical models in presence of additional food.

1. Introduction

Understanding the dynamic relationship between predator and prey and their complex properties are the central goal in ecology [1–4] due to its universal existence and importance in nature [5–8]. Ecological systems are characterized by the interactions over a wide range of spatial and temporal scales between species and their natural environment. Study of pattern formation in nonlinear complex systems has applications in natural, social,and technological sciences [2]. A well known mechanism of diffusion driven instability was first discovered by Turing [9] in 1952 in his seminal paper on the foundation of morphogen pattern formation in a diffusive chemical reaction environment. Diffusion has been observed as causes of the spontaneous emergence of ordered structures, called patterns, as related to the occurrence of what Turing called a diffusion-driven instability. Segel and Jackson [10] used reaction diffusion model to explain pattern formation in ecological context based upon the seminal work by Turing [9]. After the pioneering works of Segel and Jackson [10] pattern formation in variety of spatio-temporal models were reported [3,4,11]. Similar ideas were used to explain spatiotemporal pattern formation [2,12–16] in ecological systems.

Most of the spatiotemporal pattern occurs due to Turing instability. Different types of Turing-patterns, namely, cold spot pattern, hot spot pattern and labyrinthine pattern depending upon the choice of parameter values within

* Corresponding author. Tel.: +91 9477214234.
E-mail address: santu.ghorai0@gmail.com, swarup_p@yahoo.com (S. Poria).

http://dx.doi.org/10.1016/j.chaos.2016.01.013
0960-0779/

Turing domain were reported [3,17]. The fundamental idea of Turing mechanism tells that locally stable systems can be destabilized by diffusion, yielding a spatially nonuniform steady state or Turing pattern. Since Turing instabilities result from the interaction of reaction and diffusion, the patterns that arise are sensitive to the overall scale of the system. Recently the focus has shifted to study the non-Turing patterns [2,4] which results spatiotemporal chaos, wave of chaos and patterns near codimension two Turing–Hopf bifurcation point [18]. It is an established fact that formation of spontaneous spatiotemporal pattern is an intrinsic characteristic of predator–prey interaction [3,18,19].

Additional food is an important component of most predators diet although they receive less attention than prey in the scientific literature [20]. Additional foods fundamentally shape the life histories of many predator species. The availability of suitable additional food (non-prey food) in a predator–prey system can have significant impact on the spatiotemporal dynamics of the system. The consequences of providing additional food to predator on the spatiotemporal dynamics of predator–prey system is very useful in biological control (such as species conservation and pest management)[20,21]. In recent years, many biologist, experimentalists, and theoreticians investigated the consequences of providing additional food to predators in predator–prey systems [21–25]. But no investigation was done on the effects of additional food on the spatiotemporal dynamics of predator–prey systems. Since the presence of additional food can modify the prey–predator interaction therefore it can play a vital role in the pattern formation in an ecosystem. These facts motivates us to study a simple reaction–diffusion model with two species in presence of constant additional food for predators.

In this paper, a simple reaction–diffusion model in presence of constant additional food for predator is proposed. The additional food is characterized by its quality and quantity. The functional response of is chosen as Beddington–Deangelis type functional response [26,27]. Considering the effects of diffusion on the stability of the spatially uniform steady-state solution in the two-dimensional square domain. The existence of non-Turing pattern formation in the model is shown analytically. The spatiotemporal dynamics of the model are classified based on the dispersion relation. In presence of diffusion the model dynamics has pure-Hopf and Hopf–Turing in some parameter region. A series of numerical simulations are performed to illustrate the existence of different spatiotemporal patterns. The special attention is given to show the variation of patterns depending upon the variation of quality and quantity of additional food.

Controlling chaos in high-dimensional systems and spatiotemporal chaos especially is a very important problem with numerous applications to reaction–diffusion systems [28]. In this work, it is shown that spatiotemporal chaos can be controlled by supplying additional food.

The paper is organized in the following manner. In Section 2 the predator–prey model in presence of spatial diffusion is introduced with additional food for predators. In Section 3 the model equilibrium points are determined and their stability analysis were done without the diffusion term. In Section 4 the model is analyzed in presence of diffusion. In Section 5 the computational scheme is discussed briefly and numerical simulation results are analyzed. Finally, in Section 6 conclusion is drawn.

2. Mathematical model

A two species predator–prey model in presence of constant additional food was proposed by Srinivasu et al. [29]. Incorporating the spatial diffusion term in the model we obtain the following model

$$\begin{aligned}\frac{\partial N}{\partial T} &= rN\left(1-\frac{N}{K}\right)-\frac{e_1NP}{1+e_1h_1N+e_2h_2A}+D_1\nabla^2N\\ \frac{\partial P}{\partial T} &= \frac{n_1e_1NP+n_2e_2AP}{1+e_1h_1N+e_2h_2A}-mP+D_2\nabla^2P.\end{aligned} \quad (1)$$

Here N is the biomass of prey, P is the biomass of predator and h_1 (h_2), e_1 (e_2), n_1, (n_2) respectively represent the handling time of the predator per unit quantity A of prey (additional food), ability for the predator to detect the prey (additional food) and the nutritional value of the prey (additional food). Assume that the additional food denoted by 'A' distributed uniformly in the space. $D_1 > 0$ and $D_2 > 0$ are the diffusion coefficients. $\nabla^2 = \frac{\partial}{\partial X^2}+\frac{\partial}{\partial Y^2}$ is the Laplacian operator. In the ecological system diffusion is generally balance the species i.e, high concentration of species in a certain region will flow to another low concentrated region. We defining $C=\frac{1}{h_1}$, $b=n_1c$, $\eta=\frac{n_2e_2}{n_1e_1}$, $\alpha=\frac{n_1h_2}{n_2h_1}$ to reduce the number of free parameters. Clearly α is directly proportional to handling time h_2 of the additional food and inversely proportional to its nutritional value n_2 in other word it is inversely proportional to the quantity of additional food. The above relation transform the model (1) into the following form,

$$\begin{aligned}\frac{\partial N}{\partial T} &= rN\left(1-\frac{N}{K}\right)-\frac{CNP}{a+\alpha A+N}+D_1\nabla^2N\\ \frac{\partial P}{\partial T} &= \frac{b(N+\eta A)P}{a+\alpha A+N}-mP+D_2\nabla^2P.\end{aligned} \quad (2)$$

Now defining

$$x=\frac{N}{a},\ t=rT,\ y=\frac{CP}{ar},\ \tilde{X}=\sqrt{r}X,\ \tilde{Y}=\sqrt{r}Y$$

we obtain the following nondimensional systems,

$$\begin{aligned}\frac{\partial x}{\partial t} &= x\left(1-\frac{x}{\gamma}\right)-\frac{xy}{1+\alpha\xi+x}+D_1\nabla^2x\\ \frac{\partial y}{\partial t} &= \frac{\beta(x+\xi)y}{1+\alpha\xi+x}-\delta y+D_2\nabla^2y.\end{aligned} \quad (3)$$

where

$$\gamma=\frac{K}{a},\ \beta=\frac{bt}{r},\ \delta=\frac{m}{r},\ \xi=\frac{\eta A}{a}.$$

The parameter ξ represent the quantity of additional food supplied to predator. γ is the carrying capacity of prey which is nothing but the logistic growth rate of prey species , β is the intrinsic growth rate of predator, δ is starvation rate of predator. In the next section we analyze the model in absence of diffusion.