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Suitable or optimal noise benefits in signal detection

Shujun Liu*, Ting Yang, Mingchun Tang, Pin Wang, Xinzheng Zhang

College of Communication Engineering, Chongqing University, Chongqing 400044, China

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ABSTRACT

We present an effective way to generate the suitable or the optimal additive noises which can achieve the three goals of the noise enhanced detectability, i.e., the maximum detection probability (P_D), the minimum false alarm probability (P_{FA}) and the maximum overall improvement of P_D and P_{FA}, without increasing P_{FA} and decreasing P_D in a binary hypothesis testing problem. The mechanism of our method is that we divide the discrete vectors into six intervals and choose the useful or partial useful vectors from these intervals to form the additive noise according to different requirements. The form of the optimal noise is derived and proven as a randomization of no more than two discrete vectors in our way. Moreover, how to choose suitable and optimal noises from the six intervals are given. Finally, numerous examples are presented to illustrate the theoretical analysis, where the background noises are Gaussian, symmetric and asymmetric Gaussian mixture noise, respectively.

1. Introduction

Noise does not always deteriorate the signal processing performance. The phenomenon of the performance of some linear systems can be enhanced by adding noise is the so-called stochastic resonance (SR). Since the concept of SR was proposed by Benzi in 1981 in order to illustrate the periodic recurrence of ice gases [1], SR is observed in years of research that an improvement of a nonlinear system output is obtained when the background noise level is increased or an additive noise is injected to the system input [1–13]. Generally, the metric of improvements achieved via noise can be measured in numerous forms, such as a decrease in Bayes risk [14–16] or the probability of error [17], an increase in output signal-to-noise (SNR) [18–20] or mutual information (MI) [21–25], or an increase in detection probability (P_D) under the constraint false alarm rate property [26–31]. SR can benefit many applications, such as the physics, biological, chemical and electronic, etc [32,33].

The SR phenomenon has been studied for hypothesis testing or detection problems in recent researches. In [9], the performance of the optimal detectors, including Bayesian, minimum error-probability, Neyman–Pearson, and minimax detectors, can be improved (locally) with a nonlinear signal-noise mixture where a non-Gaussian noise acts on the phase of a periodic signal. The output performance of some suboptimal detectors can also be improved by adding additive noise or adjusting the noise level to system input according to the Minimax [14], restricted Bayesian [15], and Neyman–Pearson [27,28] criteria. The optimal noise to minimize the Bayes risk with certain constraints on the conditional risk is investigated according to the restricted Bayesian criterion in [15]. Compared [14] and [15], it is clearly that the noise enhanced detection problem in Minimax framework is a special case of the restricted Bayesian framework.

In order to increase the detection probability under the constraint on the false alarm probability (P_{FA}) in the Neyman–Pearson framework, numerous scholars and researchers have tried continuously to explore the optimal

* Corresponding author. Tel.: +86 13436044455; fax: +86 0236510 3544.
E-mail address: liusj@cqu.edu.cn, liusj03@126.com (S. Liu).

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solution and made remarkable contributions. For example, Steven Kay proved that detectability can be improved by adding independent noise to the received observation under certain conditions [26]. On this basis, a mathematical framework to investigate the noise enhanced effect on suboptimal detector is developed according to Neyman–Pearson criterion in [27], i.e., how to maximize P_D under the constraint of $P_{FA} \le \alpha$. Sufficient conditions of the detection probability can or cannot be increased are deduced, and it is determined that the optimal additive noise to maximize detection probability is a randomization of at most two discrete vectors, which is a great significance conclusion. However, Patel pointed out that there is a little defective for the proof of the optimal noise form in [30]. An optimal mathematical framework is researched in [30] for the same problem, which also proves the conclusion that the optimal noise consists of no more than two mass points. Patel and Kosko defined two sets of the additive noises according to the relationship between α and P_{FA} obtained by adding a discrete vector as noise, and found a near-optimal noise which can reach the maximum P_D as much as possible while $P_{FA} \le \alpha$.

In this paper, under two constraints that $P_{FA} \le \alpha$ and $P_D \ge \beta$, we not only consider the maximization of P_D, but also focus on the minimization of P_{FA} and the maximization of the overall improvement of P_D and P_{FA}, which is a topic worthy of study and has practical significance. Such as the maximization of the minimum number of Byzantine is equivalent to the maximization of the overall improvement of P_D and P_{FA} of the honest operating point in [34]. Different from the two sets as divided in [30], the discrete vectors are divided into six intervals in this paper according to the relationship between α and P_{FA}, and the relationship between β and P_D, where P_{FA} and P_D are obtained by adding a discrete vector as noise. It is worth to note that the information used in [30] and our method of dividing sets is the same. In fact, our method subdivides each set in [30] into three subintervals. Obviously, we all need to obtain all pairs of (P_D, P_{FA}) achieved by adding each discrete vector, and these information can be used more effectively and thoroughly to some extent in this paper. Firstly, according to the definitions of six intervals, each discrete vector can be judged as available, partial available, or unavailable for the detection result. Secondly, the available and/or the partial available discrete vectors from different intervals can be taken to make a convex combination to achieve the performance we need. Actually, our method is written based on this mechanism. As a result, the detection performance obtained by adding the randomization of the discrete vectors from different intervals is analyzed and the corresponding the ways and the conclusions to acquire the optimal additive noises for the three goals are provided. More remarkably, the suitable additive noise to enable $P_{FA} \le \alpha$ and $P_D \ge \beta$ are formulated and an explicit expression for improvability or nonimprovability is given. In addition, $\beta = P_D^x$ and $\alpha = P_{FA}^x$ are taken for the example to illustrate our idea in the following sections, where P_D^x and P_{FA}^x are the detection and the false alarm probabilities of the original detector, respectively. The main contributions of this paper can be summarized as follows:

- Six intervals of additive noises divided according to the two constraints on the detection and false alarm probabilities.
- Derivation of the suitable additive noise to meet the two constraints on the detection and false alarm probabilities.
- Formulation of an explicit expression of the suitable noise for improvability or nonimprovability.
- Determination of the optimal noises to minimize P_{FA}, maximize P_D and maximize the overall improvement of P_D and P_{FA}, respectively.

Furthermore, the method in this paper can be applied in some specific physical environment. For example, in order to exploit the escape time of a Josephson junction to detect a sinusoidal signal embedded in noise [35], N-dimensional escape times can be regarded as the observations and the probability density functions (pdfs) under two hypotheses can be retrieved by utilizing various nonparametric statistical techniques such as the kernel density estimation (KDE). The minimization of the miss probability at a fixed false-alarm probability α is equivalent to the maximization of P_D under the constraint that $P_{FA}^y \le \alpha$. Thus, the optimal additive noise, which is used to modify the escape times, to minimize the miss probability at a fixed false-alarm probability can be obtained directly by using the algorithm to maximize P_D in this paper. In addition, we also can utilize the method proposed in this paper to search the optimal noise to obtain the minimum false-alarm probability under a constraint on the miss probability. As a result, a better ROC can be realized by adding additive noise. In general, the detection and the SNR are not uncorrelated problems, and a better ROC corresponds an increase of the SNR. Thus, we also can consider how to utilize an additive noise to increase the SNR with a method similar to this paper. For instance, a superconducting quantum interference device (SQUID) is usually an electronic device to convert an applied magnetic flux into a voltage signal [36–38]. When the applied magnetic flux is treated as the input signal, we can add a direct current (DC) signal as an additive noise to the input and obtain the output voltage signal corresponding to each DC signal. Then the corresponding noise modified SNR can be calculated and the optimal additive noise is the DC signal corresponding to the maximum the noise modified SNR.

The remainder of this paper is organized as follows. In Section 2, a binary hypotheses testing problem formulation is given and the three optimization goals to maximize P_D^y, minimize P_{FA}^y and maximize the overall improvement of P_D^y and P_{FA}^y under the constraints that $P_D^y \ge P_D^x$ and $P_{FA}^y \le P_D^x$, where P_D^y and P_{FA}^y are the detection probability and the false alarm probability of the modified detector by adding additive noise. In Section 3, the form of the optimal noise for the three optimization goals are derived. Then, in Section 4, the suitable noises for the two constraints and the optimal noises for the three goals are deduced, and the sufficient conditions whether the detection performance can or cannot be enhanced are given. Further, how to determine the six intervals in practice and the noise enhanced detection under different classical criteria are analyzed too in this section. Further, numerical examples and