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A dynamic Stackelberg duopoly model with different strategies

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ABSTRACT

In this paper, a duopoly Stackelberg model of competition on output is formulated. The firms announce plan products sequentially in planning phase and act simultaneously in production phase. For the duopoly Stackelberg model, a nonlinear dynamical system which describes the time evolution with different strategies is analyzed. We present results on existence, stability and local bifurcations of the equilibrium points. Numerical simulations demonstrate that the system with varying model parameters may drive to chaos and the loss of stability may be caused by period doubling bifurcations. It is also shown that the state variables feedback and parameter variation method can be used to keep the system from instability and chaos.

1. Introduction

An oligopoly has a market structure in which a trade is completely controlled by only a few number of firms in the market producing the same or homogeneous products [1]. The dynamic of oligopoly game is more complex because firms must consider not only the behaviors of the consumers, but also the reactions of the competitors. The earliest model introduced by Cournot, in 1838 [2], gives a mathematical description of the competition in a duopolistic market. Significant additions to the theory were made exactly one hundred years later by H. von Stackelberg [3]. Expectation plays an important role in modeling economic phenomena. A firm choose its expectation rule among many available techniques to adjust its output strategy to maximize the expected profit. Naive, adaptive, bounded rationality and local approximation expectations are only a few examples. Recently, it has been shown that even oligopolistic markets may become chaotic under certain conditions. In the literature on oligopoly games, most works focus on games with homogeneous strategies, that is, players who adopt the same expectation rule [4–14]. Bounded rationality assumed in the marginal profit method is related to all producers in the models considering homogeneous expectation [10,11]. Another branch of literature is made up of studies in which games with heterogeneous players are taken into consideration [15–29]. The assumption of players adopting heterogeneous rules to decide their production is more realistic than the opposite case [28,29]. In fact, among a practically infinite number of ways of being boundedly rational, it is quite rare that two firms behave according to the same rules. Ref. [19] used the technique of Agiza and Elsadany [15,16] to analyze a duopoly game with heterogeneous players and nonlinear cost function. Ref. [20,21] studied a duopoly game with heterogeneous firms assuming a micro-founded nonlinearity on the demand function. Ref. [27–29] studied the dynamics of oligopoly models with more heterogeneous firms.

Most of the previous works are based on the Cournot model and the modifications to discuss the complex

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dynamics. There is little literature dealing with Stackelberg model and the modifications in studying dynamical behaviors [30]. The Stackelberg games are natural models for many important applications that involve human interaction. In many applications, players facing human followers (adversaries) may deviate from their expected responses to the game theoretic optimal choice, because of their bounded rationality and limited observation [31]. Thus, a human adversary may cause an unacceptable degradation in the leader's reward [32].

The main purpose of this study is to formulate a duopoly Stackelberg game with different strategies and investigate its dynamical behaviors and chaos control. The leader firm chooses strategic variable first, then the follower firm chooses strategic variable. In the subsequent stages, two firms update their strategies in order to maximize their profits in the market. In this Stackelberg game each firm tries to maximize its profit according to local information of its strategy. We consider that each player forms a different strategy in order to compute its expected output. We assume that the leader represents a boundedly rational player and the follower has local approximation expectations.

The rest of this paper is arranged as follows. In Section 2, a duopoly Stackelberg game is briefly described. A duopoly Stackelberg game model with different strategies has been formulated. In Section 3, we study the fixed points and the dynamics of the model, showing explicit parametric conditions of the existence and local stability of the equilibrium. The local stability analysis of the equilibrium are developed. The dynamics for a duopoly Stackelberg game model with different strategies are analyzed. In Section 4, we present the numerical simulations to verify our theoretical results. In Section 5, we exerted control on the duopoly Stackelberg game model. Finally, some remarks are presented in Section 6.

2. The model

2.1. The duopoly Stackelberg model

The classic Stackelberg game is divided into two stages. In stage 1, the*planning phase*, each player chooses strategies, and concludes forward contracts for output. In stage 2, the *production phase*, they choose the quantities to be produced. The players act sequentially in planning phase, act simultaneously in production phase, and the choices made in stage 1 are common knowledge in stage 2.

We consider a duopoly Stackelberg game. Two firms, labelled by $i = 1, 2,$ produce the perfectly substitutive goods for sale in the market. Firm 1 is the Stackelberg leader and firm 2 is the follower. The strategy space is the choice of the output. The decision-making takes place in the discrete time periods $t = 0, 1, 2, \ldots$. Let $q_i(t) > 0$ represents the output of ith firm during period t, with a production cost function $C_i(q_i)$. The price prevailing in period t is determined by the total supply $q(t) = q_1(t) + q_2(t)$ through a demand function $p = f(q)$. In this model the demand function is assumed linear, which has the form $f(q) = a - bq$, where a and b are positive constants.

Let Q_i be the announced plan products of the ith firm. There is a difference between the announced product and the actual output of ith firm during period t. We assume that the firms use different production method and the cost function is proposed in the nonlinear form

$$C_i(q_i) = c_i(q_i - Q_i)^2, \quad i = 1, 2$$

where the parameters c_i are positive shift parameters to the cost function of the ith firm. With these assumptions, the profit of the ith firm at the period t is given by

$$\Pi_i(q_1, q_2) = q_i(a - bq) - c_i(q_i - Q_i)^2, \quad i = 1, 2$$

Then the marginal profit of the ith firm at the point (q_1, q_2) is given by

$$\frac{\partial \Pi_i}{\partial q_i} = a + 2c_iQ_i - 2(b + c_i)q_i - bq_j, \quad i = 1, 2, \ i \neq j \quad (1)$$

Let the partial derivative of Π_i respect to q_i equal to zero. We can obtain the equilibrium solution for the firms [30].

$$\begin{cases} q_1 = \dfrac{a(b + 2c_2) + 4(b + c_2)c_1Q_1 - 2bc_2Q_2}{3b^2 + 4b(c_1 + c_2) + 4c_1c_2} \\ q_2 = \dfrac{a(b + 2c_1) + 4(b + c_1)c_2Q_2 - 2bc_1Q_1}{3b^2 + 4b(c_1 + c_2) + 4c_1c_2} \end{cases} \quad (2)$$

Take Eq. (2) into Π_2. We calculate a derivative of Π_2 with respect to Q_2 and set it to zero for the optimal action of firm 2. This optimization problem has unique solution in the form

$$Q_2 = \frac{4(b + c_1)(b + c_2)[a(b + 2c_1) - 2bc_1Q_1)]}{b[b(3b + 4c_1)^2 + 8(b + c_1)(b + 2c_1)c_2]} \quad (3)$$

Take Eqs. (2) and(3) into Π_1, differentiate Π_1 with respect to Q_1. Equating the partial derivative to zero, we can obtain the optimal action of firm 1 in the form

$$Q_1 = \frac{4a(b + c_1)(\Delta_1 - bc_2)(\Delta_1 - 2bc_2)}{b\Delta} \quad (4)$$

where $\Delta_1 = 3b^2 + 4b(c_1 + c_2) + 4c_1c_2$, and $\Delta = (9b + 8c_1)\Delta_1^2 - 24b(b + c_1)c_2\Delta_1 + 16b^2(b + c_1)c_2^2$. Take Eq. (4) into Eq. (3) and the simplified form is

$$Q_2 = \frac{a(b^2 + \Delta_1)[(3b + 2c_1)\Delta_1 - 4b(b + c_1)c_2]}{b\Delta} \quad (5)$$

Substituting Eqs. (4) and(5) into Eq. (2), we obtain the equilibrium solution for the firms in the Stackelberg game [30].

$$\begin{cases} q_1^* = \dfrac{a[(3b + 4c_1)\Delta_1 - 4b(b + c_1)c_2](\Delta_1 - 2bc_2)}{b\Delta} \\ q_2^* = \dfrac{a[(3b + 2c_1)\Delta_1 - 4b(b + c_1)c_2]\Delta_1}{b\Delta} \end{cases} \quad (6)$$

From Eq. (6), we can easily verify that $q_1^* > 0$ and $q_2^* > 0$.

2.2. Duopoly Stackelberg game model with different strategies

We assume different expectations: i.e. firm 1 is boundedly rational and firm 2 is local approximation. The boundedly rational firm 1 has no complete knowledge of the market. Firm 1 tries to use local information based on the marginal profit $\frac{\partial \Pi_1}{\partial q_1}$. It decides to increase (decrease) its