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Discrete time population dynamics of a two-stage species with recruitment and capture

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ABSTRACT

This work models and analyzes the dynamics of a two-stage species with recruitment and capture factors. It arises from the discretization of a previous model developed by Ladino and Valverde (2013), which represents a progress in the knowledge of the dynamics of exploited populations. Although the methods used here are related to the study of discrete-time systems and are different from those related to continuous version, the results are similar in both the discrete and the continuous case what confirm the skill in the selection of the factors to design the model. Unlike for the continuous-time case, for the discrete-time one some (non-negative) parametric constraints are derived from the biological significance of the model and become fundamental for the proofs of such results. Finally, numerical simulations show different scenarios of dynamics related to the analytical results which confirm the validity of the model.

1. Introduction

Each year the human race captures approximately 40% of the planet's primary production for its own profit, destroying over 35% of terrestrial net primary productivity [20]. In the last 20 years, the planet has lost a million square kilometers of Arctic ice and two million square kilometers of forests, the concentration of CO_2 has increased by 10% and over 20,000 species have become extinct [12]. Faced with this dramatic situation, different international organizations are trying to stop the looting occurred and posing different ways to resolve the problem.

In this context, issues like the conservation and sustainability of exploited species are being studied in various areas of knowledge. Recently, different mathematical models for population dynamics with harvesting have been developed in applied mathematics, such as those by Zhang et al. [22], Song and Chen [17], Jing and Ke [10], Srinivas et al. [19], Chakraborty et al. [5], Pal et al. [16], Ladino and Valverde [11], Atehortua et al. [1], Fresard and Ropars-Collet [7], Manna and Samanta [14], Yuan et al. [21]. However, many of these models are of theoretical character and do not show their applicability to real situations.

Recent works proposed by Ladino and Valverde [11] and Atehortua et al. [1] have been developed inspired by empirical considerations and with the objective of proposing possible solutions to real problems. However, several of these problems still remain open and their study could lead to a better approximation of the real dynamics of the species modeled and to the formulation of policies to help their conservation.

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Discretization of nonlinear continuous dynamical systems providing mapping structures which can be employed to analytically predict periodic orbits and the corresponding stability and bifurcations, as affirmed by Luo [13], is an interesting and prominent trend. In this sense, the present work deals with the study of a discrete-time version of the model for the dynamics of a two-stage species with recruitment and capture proposed by Ladino and Valverde [11]. More in detail, the main purpose of this work is to provide and analyze the dynamics of a population with a two-stage structure (i.e. two subpopulations): a pre-recruit stage (eggs, larvae and juveniles) and an exploitable stage (recruit or adult). The model examines the important role of pre-recruit population, which is usually not visible to fishing activity, in the dynamics of the exploitable population. Such a study is of great interest for biological applications, since it investigates conditions on parameters under which the survival of the species considered is preserved in the long term and can consequently suggest policies to prevent its extinction.

In recent years, researchers have paid more attention to discrete-time models, especially in population dynamics. Some of the reasons which explain this fact are: first, since statistical data on population size are collected in discrete time, it is more convenient and accurate to describe a two-stage species model by using a discrete-time model rather than a continuous-time one; second, we can get more accurate numerical simulation results by using discrete-time models; finally, the numerical simulations of continuous-time models are actually obtained by discretizing the models. These reasons illustrates the appropriateness and interest of our study.

Although the methods used here are related to the study of discrete-time systems [9] and are different from those related to continuous version, the similarity of the results confirms the skill in the selection of the factors to design the model, and indicate us that the absence of contradictions between a continuous model and its discretization one should be taken into account. In fact, we demonstrate that $\mathbb{R}^2_+$ continues to be a positively invariant set for the system, under some necessary parametric constraints, and identical equilibriums with the same kind of stability are found, also exhibiting a transcritical bifurcation when a threshold parameter $\mathcal{R}_0$, which depends on the parameters of the model, passes through its singular value $\mathcal{R}_0 = 1$. Nevertheless, for the discrete-time model some necessary (non-negative) parametric constraints emerge from the biological significance of the model and become fundamental for the proofs we perform of such results. This novelty alerts us that when modeling with discrete-time systems additional cautions have to be considered.

The study presented is of great interest in biological applications since it investigates conditions under which the survival of the species in the long term is preserved, and consequently policies to avoid extinction can be suggested. It represents an initial attempt to model the dynamics of exploited populations in discrete time and, hence, it can immediately be compared to the continuous time version. Furthermore it opens a way to stimulate and strengthen the debate between biologists and mathematicians on new formulations to obtain more realistic frameworks.

The paper is organized as follows. In Section 2, we describe the model of population dynamics in discrete time and establish the necessary conditions to have a biologically coherent formulation of the model. In Section 3, we deal with the question of the existence and stability (local and global) of fixed points. More precisely, we pursue conditions such that the extinction equilibrium is globally asymptotically stable. In Section 4, we illustrate numerical simulations showing different scenarios of possible dynamics of the model.

2. The model

The model herewith proposed describes the dynamics of a species in which the population has a two-stage structure and the relationship between the two stages is based on recruitment.

In biology, recruitment refers to survival of the individuals of a species through growth until they become part of the breeding population. In the case of fish, the life cycle develops through the stages of egg, larva, juvenile and adult. In its early life a fish, does not usually get captured during fishing activities, because it is too small or it remains outside fishing areas. However, when it grows, conditions are become different and, due to changes in its size or location, the fish can be detected and caught with one of the different fishing methods available. This change of state which makes the fish become detectable to fisheries for the first time is also known as recruitment. In this sense, it is said that a fish passes from the pre-recruit phase to the post-recruit phase and becomes integrated into the exploitable population for the first time. Of course, in fisheries, it is interesting to know the exploitable fish population because it is the part of the total population visible for fishing [6], but the change of state from pre-recruit to post-recruit is not important only for the biological life cycle of a specie, but also for its exploitation. Actually, for some species, young fishes (or pre-recruits) occupy nursery grounds where adults are not present in an appreciable quantity; in such a case, the recruitment implies a migration to exploitation area from the nursery grounds (Beverton and Holt [2]).

We consider the concept of stock as a well-defined subset of a given species, whose population parameters remain constant in the distribution of the stock [18]. In the model, the stock is the number of individuals or population size.

The assumptions and population parameters characterizing the population dynamics modeled are described below.

1. The total population at the time $t \in \mathbb{N}$ is divided into two subpopulations: *pre-recruit population* $x(t)$ and *exploitable population* $y(t)$, $x(t) \geq 0$ and $y(t) \geq 0$.
2. The population evolves in discrete time. Therefore, the model is represented by a system of nonlinear difference equations.
3. The *reproductive rate* of the exploitable population is $\delta > 0$. It is also assumed that only adult individuals are mature enough to breed.