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Nonlinear oligopolistic game with isoelastic demand function: Rationality and local monopolistic approximation

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ABSTRACT

Isoelastic demand function have been used in literature to study the dynamic features of systems constructed based on economic market structure. In this paper, we adopt the so-called Cobb–Douglas production function and study its impact on the steady state of an oligopolistic game that consists of four oligopolistic competitors or firms. Briefly, the paper handles three different scenarios. The first scenario introduces four oligopolistic firms who plays rational against each other in market. The firms use the myopic mechanism (or bounded rational) to update their production in the next time unit. The steady state of the obtained system in this scenario, which is the Nash equilibrium, is unique and its characteristics are investigated. Based on a local monopolistic approximation (LMA) strategy, one competitor prefers to play against the three rational firms and this is illustrated in the second scenario. The last scenario discusses the case when three competitors use the LMA strategy against a rational one. For all scenarios discrete dynamical systems are used to describe the game introduced in all scenarios. The stability analysis of the Nash equilibrium is investigated analytically and some numerical simulations are used to confirm the obtained analytical results.

1. Introduction

Literature has shown that researchers have been interesting in the dynamic characteristics which arise among firms involved in competed economic market. The importance of such competition attracts many researchers to study in depth the chaotic behaviors appear in this competition. The studying and investigation of this type of competition have lead to imperatively use the game theory on modeling and studying the competition in details. The theory of game has been widely recognized as an authoritative tool in many fields such as economy. For the best of our knowledge, eleven game-theoretic scientists have won the Nobel Prize in economics.

Cognitive and computational skills are required for those firms competed in an oligopolistic game. Knowing such skills may help the oligopolist to properly determine the curve of demand related to the produced quantity and consequently expect its oppenent's production in the next period of time. Once the demand curve is available, a one period problem of optimization is constructed and hence be ready to be solved by the firms. Several studies have been created to investigate the important of rationality [1–6]. They have concluded that reducing the rationality that is resulted in due to cognitive and computational skills have leaded to some complex behaviors of the competed firms. In [25], the authors have set up a Bertrand duopoly game based on differentiated products and they have come

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up with conclusion of the existence of complex behavior of that game. The dynamic behavior of a delayed bounded rationality duopoly game has introduced and studied in [26]. Using nonlinear cost function, the dynamic characteristics of a Cournot duopoly game have been investigated in [27].

Cobb–Douglas production function has been introduced in 1927 and since now it has been used to study certain behaviors of Cournot duopolistic games. For example, this function has been accompanied with a unimodel function to study complex characteristics of a duopolistic model [7]. Other studies have paid much more attention to some of the important adjustment mechanisms like bounded rationality and LMA (local monopolistic approximation) mechanisms. Rationality mechanism has been intensively used in literature [8,9] and it needs only from the firms few information about the market structure in order to be able update their production in the next time step based on the change occurring in the marginal profit. Indeed, knowing the demand and cost functions are not compulsory to be identified by the firms but firms should be more concentrated on any change happens in the market when small changes of the quantities produced due to estimating the marginal profit. More information and knowledge on the rationality mechanism can be found in literature [8–10]. In [11], the author has introduced the so-call local monopolistic approximation (LMA), After that Naimzada and Sbragia [4] have adopted this mechanism in their model and have studied its influences on the behavior of the model. The mechanism has been applied also by Naimzada and Ricchiuti in [12]. Firms apply LMA mechanism are not requested to know any information or knowledge regarding to demand function of the market. Although the demand function is unknown, the firms (competitors) claim that it is linear and therefore they estimate this linear function using small information available about the current state of the market in terms of quantities produced, prices and the true demand curve.

New directions of the application of evolutionary game theory in continuum and networks have been introduced in [15,16]. In [16], an extension of evolutionary games has been applied on multilayer networks on which two-layer scale-free networks with degree mixing are considered. Wang and Perc [17] have discussed high-enough evolutionary fitness of individual players in the form of additional links that bridge the gap between two initially disconnected populations. Aspiration to the fittest based on the so-called prisoner's dilemma has been introduced by Wang and Perc [18]. Assuming cooperation among players, Wang et al. have studied the influence of aspirations on that cooperation in [19]. The influence of population density on public cooperation has been investigated by Wang et al. [20]. In [21], the replicator equation has been used to study public goods game in well-mixed populations and using Monte Carlo simulations. The evolution of cooperation in games that describe social dilemmas has been studied by Perc et al. [22]. Interested reader are advised to follow [23,24] for information on cooperation and evolutionary games.

The current paper introduces an oligobolistic model consisting of four competed firms. Only few studies have dealt with such models in literature and in all those studies only rationality is the only mechanism that has been used to investigate the complex characteristics of them. In addition, Naive strategy has been incorporated in [13] with bounded rationality to build a heterogeneous duopolistic model. Delaying with oligopolistic models that have four competed firms is difficult than those dealing with two firms. Here, we investigate the stabilization of LMA when it is combined with bounded rationality and an economic isoelastic function is used. Introducing such heterogony among players is the main purpose in this paper and does not handled before in four firms models in literature. The obtained results show that the considered models exhibit a flip bifurcation except for the case of four LMA firms. In addition, we show that the stability of those models are governed not only by the degree of rationality but also by marginal costs.

To ease reading the current paper, it is divided into two parts. In the first part, we introduce an oligopolistic game consisting of four firms (or competitors). Each firm in this game use the rationality strategy to dynamically update their productions where the decision variables here are represented by the quantities produced by each firm. The second part proposes two different scenarios. One of the scenarios study an oligopolistic game on which three rational competitors play against one LMA competitor while the other scenario assumes the inverse on which three LMA competitors play versus one rational. In each part, a four dimensional system is used to the describe the game. Nash equilibrium of the game is obtained and is unique. Stability analysis of the Nash equilibrium is investigated based on the characteristics of the eigenvalues. Some numerical experiments are employed to enhance and confirm the obtained results.

The paper is outlined as follows. Section 2 introduces an oligopolistic game in which four rational firms are in competition. The steady state of the game which is Nash equilibrium is obtained and its characteristics are illustrated. Section 3 proposes an oligopolistic game where three rational firms play against one LMA firm then three LMA firms against one rational. For both the Nash equilibrium is obtained and studied in details. Finally, some conclusions to end the discussion of the current paper are given in Section 4.

2. Rational model

The general form of the production function of Cobb–Douglas when there are four firms are in competition is given by,

$$U = \Pi_{i=1}^{4} q_i^{\alpha_i}, \sum_{i=1}^{4} \alpha_i = 1 \tag{1}$$

where, q_i denotes to the quantity produced by firm i. α_i is a constant for all i and is called the output elasticity that means that a consumer uses fraction α_i of his/her wealth in purchasing good i. The formula (1) is called a utility function and each firm in the competition wants indeed to maximize this function. For this reason, we assume that there is a budget constraint governed the market, $\sum_{i=1}^{4} p_i q_i = 1$, where p_i, is the price of quantity sold in the market by firm i. With (1) and the budget constraint,